ASPECTS OF ART LECTURE

GEOMETRICAL PERSPECTIVE FROM BRUNELLESCHI TO DESARGUES: A PICTORIAL MEANS OR AN INTELLECTUAL END?

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> It is the very same taste which relishes a demonstration in geometry, that is pleased with the resemblance of a picture to an original, and touched with the harmony of musick. All these have unalterable and fixed foundations in nature, and are therefore equally investigated by reason. (Sir Joshua Reynolds)[1]

THE idea that the visual arts gained in dignity through their associations with geometry had become an academic commonplace by the time Reynolds delivered his seventh 'Discourse' at the Royal Academy in 1776. However, Reynolds himself was no geometrician, and it is doubtful if any painter of his generation was in a position to understand advanced projective geometry, as practised at that time by Monge, let alone make a positive contribution to its progress. This gap had not been apparent in the earlier stages of the Renaissance, though, as I hope to show, the development of perspective science away from the concerns of artists had become pronounced by 1600.

From the time of Brunelleschi and Alberti, perspective had given the painter his most powerful argument in promoting his vocation into the intellectual realm of the *quadrivium* of the liberal arts. Today, the nature of perspectival representation continues to play a central role in discussions of perception and representation, both in the context of experimental psychology and in relation to fashionable concerns with signifying systems in post-structuralist theory.[2] For the historian, the story of perspective reveals an

[1] Sir J. Reynolds, *Discourses on Art* (1797) (ed. R. Wark, 1965), Discourse VII, 10 December 1776, p. 97.

[2] For a review of some recent contributions to these questions see M. Kemp,

intricate nexus of contacts with the astonishing range of disciplines related to the exact science of geometry. During the course of this essay, we will be striking glancing blows on at least some of these disciplines—cosmology, astronomy, optics, cartography, architecture, military engineering, ballistics, and even stone-cutting. The place of perspective science in the Renaissance scheme of things is graphically summarized in John Dee's preface to Euclid (Pl. I). 'Perspective' stands at the head of nineteen mathematical pursuits, including the special discipline of 'Zography—which demonstrateth and teacheth how the intersection of all visual pyramids made on any plain assigned . . . may be by hues and proper colours represented'.[1]

This philosophical context for the painters' science was a source of great strength, but it was also responsible for the ultimate removal of advanced perspective theory beyond the artists' grasp. I think it is arguable that Dürer was the last professional painter to make a specifically mathematical contribution to three-dimensional geometry. And, certainly by 1600, innovations in mathematical perspective had become the prerogative of 'professional' mathematicians, such as Commandino, Guidobaldo del Monte, Benedetti, and specialists in the civil applications of mathematics, such as Simon Stevin and Girard Desargues. This professional mathematicizing of perspective stretched to breaking point a series of tensions which had been apparent from the first—tensions between theory and practice, or more particularly between men of letters and men of the brush.

A clear potential for the divergence was built into early perspective by the coming together of diverse components in its formation. The main components may be summarized as: (i) workshop practice as a series of empirical solutions to the problem of creating an effective illusion of space; (ii) a range of applied techniques of estimation and measuring from late medieval surveying; (iii) the mathematical theory of plane projection based ultimately upon Euclidian postulates; (iv) the science of optics as developed to a point of high sophistication by Alhazen, Witelo, Bacon, and Pecham in the Middle Ages.

To some extent, the science of optics came to act as an intermediary between Euclid and the observation of nature, revealing the secret order of God's creation through the geometrical office of

'Seeing and Signs: E. H. Gombrich in Retrospect', *Art History*, vii (1984), 228–43.

[1] J. Dee, 'Mathematical Preface' to Euclid (1563), published in *Euclid's Elements of Geometry* . . . etc. (ed. J. Leeke and G. Serle, London, 1651).

sight. Theorists and theoretically minded artists keenly seized upon the idea that the mathematics of art mediated between the incorporeal perfection of Euclidean geometry and material forms. It was much as Kepler found when rhapsodizing about the cosmic beauty of the six-cornered snowflake: 'From this almost Nothing, I have found the all-embracing universe itself! . . . here I am exhibiting the soul of the "thrice greatest Animal", the globe of the Earth, in the "atom" of a snowflake.'[1] Yet, in their eagerness to embrace the geometrical basis of beauty, the artists were moving on to dangerous ground. When we find both Lomazzo and Poussin paraphrasing Ficino to the effect that 'beauty is so far removed from corporeal matter that it cannot begin from matter, unless this is conditioned by the three preparations which have been stated to be incorporeal' (i.e. 'order', 'mode', and 'species'), they were in danger of betraying the very materiality of their own illusionistic craft.[2] If we agree with Witelo, the Polish student of optics, who had developed Arabian empiricism in a distinctly Platonizing direction, that the artificial beauty of geometry is far superior to 'natural things', we may wonder why we need to represent the forms of nature at all to experience beauty.[3]

By the early seventeenth century, as I hope to show, perspective theory had arrived at such a position of abstraction from the effects of nature, that it became a deductive science, founded on postulates in the Euclidian manner. This changed basis of perspective science is in one sense the story of this paper. Yet it is part of the richness of art that a few great practitioners continued to marry natural description and deductive perfection, in a way which ultimately defies the dry logic of historical analysis.

The tensions of which I have spoken were inherent in the first Renaissance book to treat the new perspective science, Alberti's *De pictura*. The author paraded half-explanations from Euclidian

[1] J. Kepler, *The Six-Cornered Snow-Flake* (1611) (trs. C. Hardie, Oxford, 1966), p. 39.

[2] M. Ficino, *Sopra lo amore o ver convito di Platone* (Florence, 1544), v, paras. 3–6, p. 94; G. P. Lomazzo, *L'idea del tempio della pittura* (Bologna, 1590), section XXVI; Poussin in G. P. Bellori, *Vite de' pittori, scultori et archittetti moderni* (Rome, 1672), p. 461, and A. Blunt, *Nicolas Poussin* (London and New York, 1967), pp. 364–5. See E. Panofsky, *Idea. A Concept in Art Theory* (trs. J. Peake, 1968), pp. 139–45 and 243, note 22, and D. Summers, *Michelangelo and the Language of Art* (Princeton, 1981), pp. 421–2. Poussin's version is somewhat freer than Lomazzo's.

[3] Witelo, *Opticae thesaurus Alhazeni . . . et . . . Vitellonis Thurinopoloni Opticae libri decem* (ed. F. Risner, Basel, 1572), iv, 148.

geometry and medieval optics, before outlining in some detail a practical means of constructing spatial illusion. He elusively failed to commit himself on the precise relationship between these factors, passing abruptly from an outline of the visual pyramid

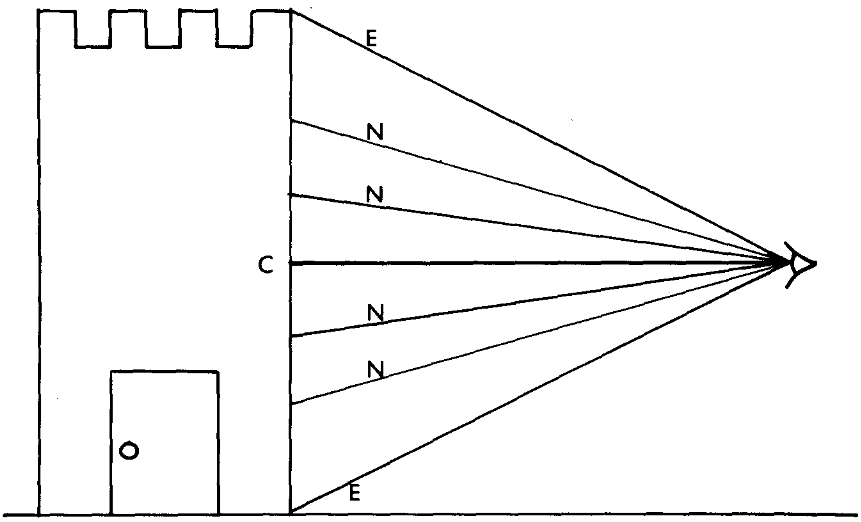


Fig. 1. Alberti's Visual Pyramid (drawn by M.K.): E, extrinsic rays (recording shape); N, intrinsic rays (recording surface); C, centric ray (for 'certification' of sight).

(Fig. 1) to his constructional procedure (Fig. 2), with no proof as to why the former results in the latter. His mechanism for checking his construction, by drawing a diagonal across his checkerboard 'floor', is related to a long-standing workshop device (Fig. 3) for the spatial organization of floor tiles. Varieties of this method were practised with great intricacy by the Lorenzetti in *trecento* Siena, taken up by Uccello in the *sinopia* of his *Nativity*, and illustrated in the early sixteenth century by Viator (Fig. 4).[1] But, again, Alberti's treatment of the diagonal device is incompletely developed, in that he fails to record the convergence of multiple diagonals to the two lateral 'vanishing points'. This so-called 'bifocal' method is a topic to which I will return. There is a comparable problem of integration in Ghiberti's third *Commentary*, which in itself contains

[1] The 'bifocal' workshop method is discussed by R. Klein, 'Pomponius Gauricus on Perspective', *Art Bulletin*, xliii (1961), 211–30. For Viator's *De artificiali perspectiva* (Toul, 1505) see W. Ivins jnr., *On the Rationalisation of Sight* (New York, 1938).

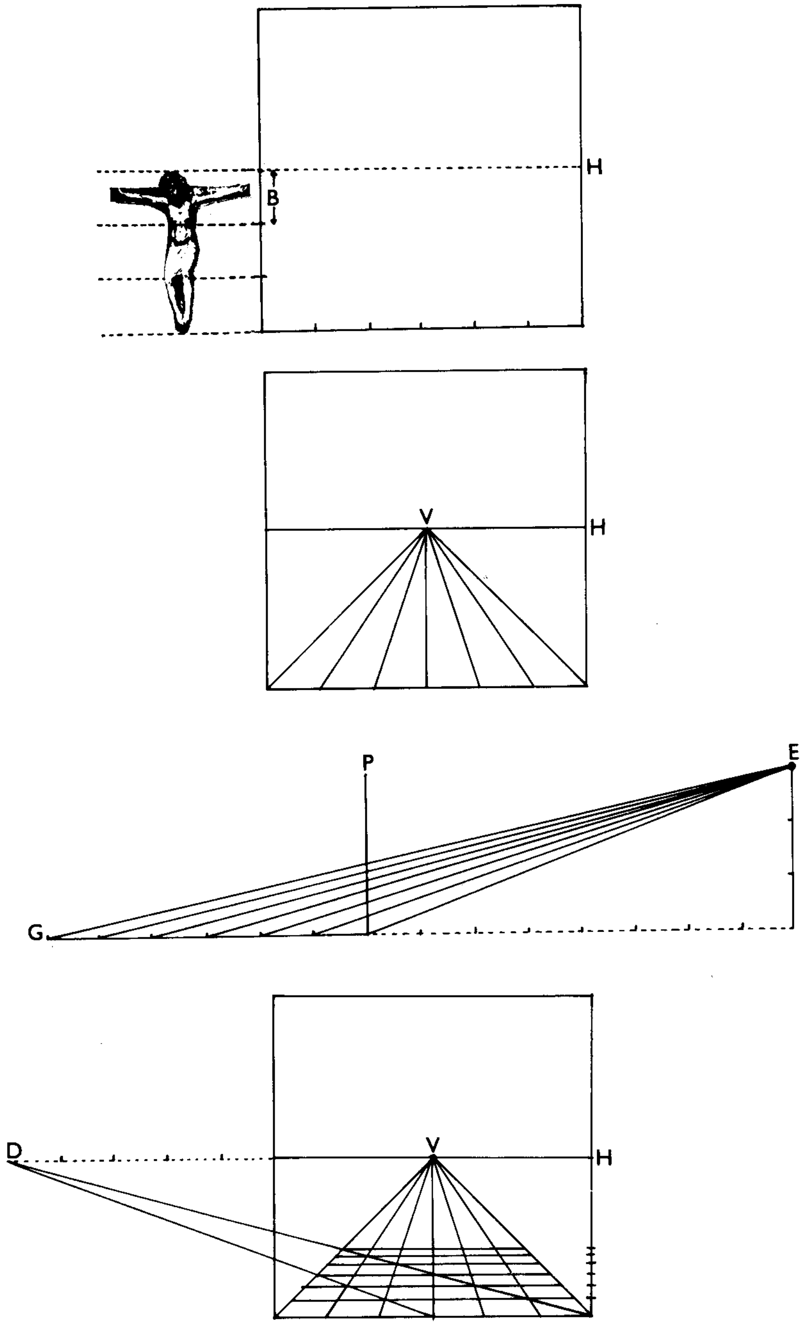


FIG. 2. Alberti's Perspective Construction (M.K.): B, *braccio* module ($\frac{1}{3}$ height of man) for division of base of picture; H, horizon line; V, point to which orthogonals converge; E, eye; G, ground plane; P, picture plane, on which are intersections for horizontals of floor tiles (for final construction below); D, lateral or 'distance' point produced by diagonals, but unacknowledged by Alberti. (Note: DV = EP.)

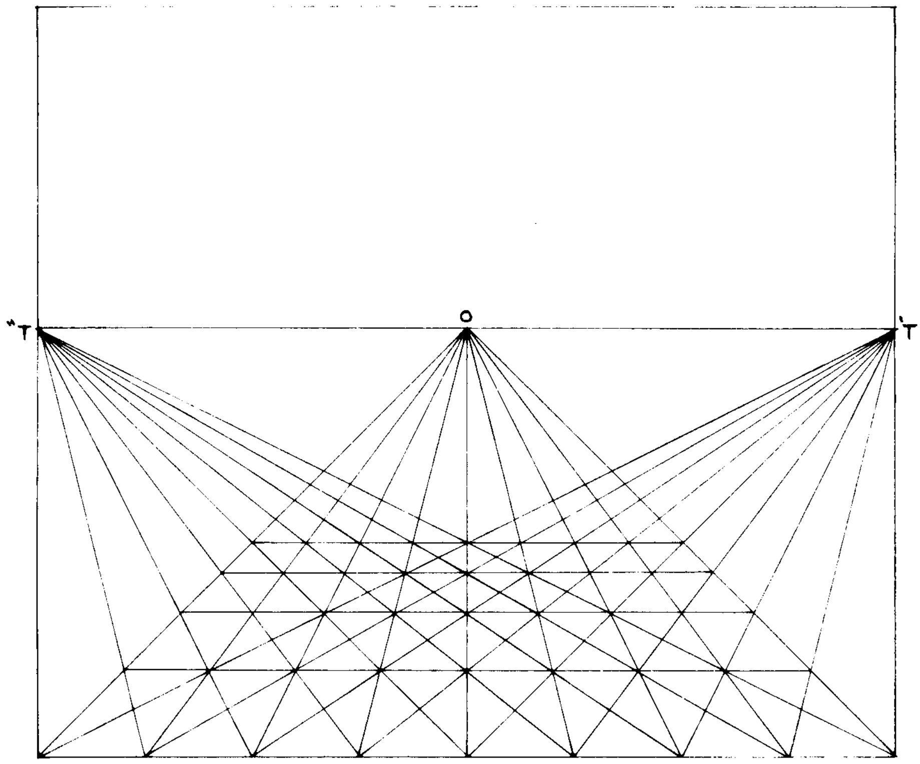


FIG. 3. 'Bifocal' Method of Perspective Construction, as in Uccello's '*Nativity*' (M.K.): T′, T″, lateral points, joined to equal divisions at base of picture (Viator's 'tiers points'); O, central point produced by lines through intersections of lines to lateral points. (Note: visual angle = 90°.)

an intelligently edited selection of relevant sections from Alhazen, Witelo, Bacon, and Pecham, but which does not ultimately demonstrate the geometrical proof of the Albertian and workshop constructions in terms of the quoted texts.[1]

Although Alberti fails to integrate the disciplines to which he looks for support, the dominant flavour of his approach is clear; it was founded upon Euclidian notions of proportionality, particularly as manifested in the properties of similar triangles. He stressed that 'all quantities parallel to the intersection remain proportional'—in other words, that the essential aspect of a perspective projection on the picture plane was that forms

[1] G. Ten Doesschate, *De Derde Commentaar van Lorenzo Ghiberti in Verband met de Middeleeuwsche Optick* (Utrecht, 1940); and J. Gage, 'Ghiberti's Third Commentary and its Background', *Apollo*, xcv (1972), 364–9.

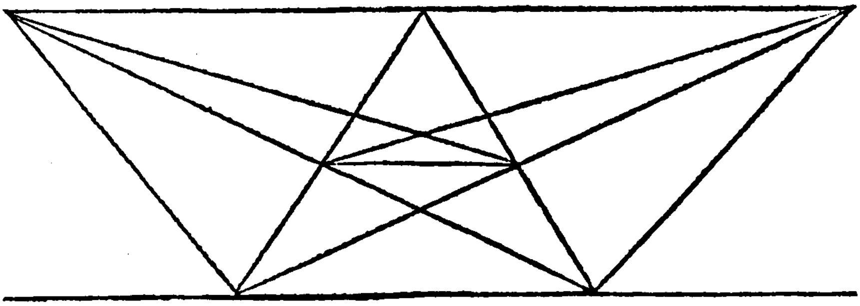


FIG. 4. Jean Pelerin (Viator), *'Bifocal' Perspective Construction*, from *De artificiali perspectiva* (Toul, 1504).

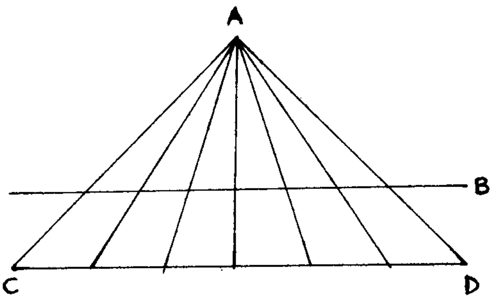


FIG. 5. Piero della Francesca, *Parallel Intersection of Seven Converging Lines* (M.K.): A, 'eye'; B, intersecting plane; CD, 'base' divided into equal spaces.

retained their proportional relationship no matter how much they were reduced in actual size.[1]

This foundation in the Euclidian geometry of triangles is even more dominantly characteristic of Piero della Francesca in *De prospettiva pingendi*. He defined the picture as 'the plane on which the eye with its visual rays marks objects proportionately', and roundly criticized fellow painters who 'do not understand the power of lines and angles'.[2] The diagram illustrating his proportional intersection (Fig. 5) possesses considerable implications in the geometrical theory of scales and ratios. Greek geometers had already demonstrated—and here I am adapting a later figure from Daniele Barbaro's *La Pratica della perspettiva* (Fig. 6)—that

[1] *Leon Battista Alberti 'On Painting' and 'On Sculpture'*, (ed. and trs. C. Grayson, London, 1972), para. 15, pp. 50–1. See R. Wittkower, 'Brunelleschi and "Proportion in Perspective"', *Journal Warburg and Courtauld Institutes*, xvi (1953), 275–91, for fine analyses of proportional perspective in Alberti, Piero and Leonardo.

[2] Piero della Francesca, *De Prospettiva pingendi* (ed. G. N. Fasola, 2 vols., Florence, 1942), I, pp. 64–5, and 128 (opening of book II).

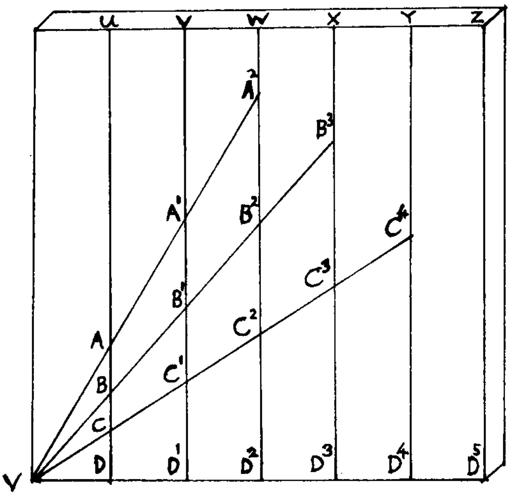


FIG. 6. Daniele Barbaro, *Ratios of Division of a Line* (M.K., adapted from *La pratica della perspettiva*, Venice, 1569). The parallel lines U, V, W, X, Y, Z are equidistant. $VA^2 = VB^3 = VC^4 = VD^5$. The lines from V are thus divided into spaces according to the ratios $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}:\frac{1}{6}$. (Note: the points A^2, B^3, C^4, D^5 describe the arc of a circle centred on V.)

proportional relationships existed both in the 'intersections' and the 'rays', such that:

$$\frac{BA}{BC}\times\frac{B'C'}{B'A'}=\frac{VA}{VC}\times\frac{V'C'}{V'A'} \text{ and } \frac{AC}{BC}:\frac{AD}{BD}=\frac{A'C'}{B'C'}:\frac{A'D'}{B'D'}, \text{ etc.}$$

Piero was awake not only to such relationships, but also to the further possibility of quantifying Euclidian proportions. Thus he explained that if an eye at A (Fig. 7) 3 *braccia* high and 10 *braccia* from the intersecting plane, P, views the point C which is 20 *braccia* behind the plane, the intersection at B will be 2 *braccia* above E, according to the proportional formula DC:EC = AD:BE. He explained that 'the second to the first line [on the intersection] is always in the same proportion as the distance from the eye to the first line is to that of the second line to the eye'—a formulation which is easier to follow if we illustrate his text for him (Fig. 8).[1] When the eye is 4 *braccia* from the intersection AB, the ratio of AB to the intersection made by CD on AB will be equivalent to 105:84; the ratio of the intersection made by CD to that made by EF will be 84:70; and the ratio of the intersection made by EF to that made by GH will be 70:60—or more simply, 5:4, 6:5, and 7:6.[2]

[1] Piero della Francesca, *De Prospettiva pingendi* (ed. G. N. Fasola, 2 vols., Florence, 1942), I, pp. 74–5.

[2] Ibid., I, p. 74.

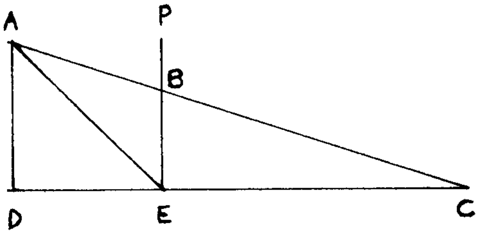


FIG. 7. Piero della Francesca, *Intersection in Relation to Viewing Height, Viewing Distance and Object Distance* (M.K.): AD, viewing height; P, picture plane; DC, distance of object from viewer; EC, distance of object behind picture plane; BE, height of intersection; DC:EC = AD:BE.

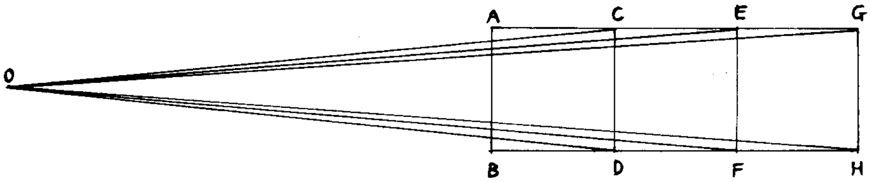


FIG. 8. Piero della Francesca, *Ratios of Intersection* (M.K.): O, eye (4 *braccia* from AB); AB, picture plane; CD, EF, GH, equal lines at *braccio* intervals, parallel to AB.

This is in keeping with his attempt to apply arithmetical calculations to the Platonic solids discussed and illustrated in his *De Quinque corporibus regularibus*.[1]

It is easy, therefore, to see how his basic perspective construction (Fig. 9) is an exercise in proportional diminution, and how it is relevant to the obsessively proportional mien of his most elaborate paintings, like the *Flagellation*. It is also easy to understand how Piero played a decisive role in the establishing of geometrical forms in a more abstract manner as suitable items in decorative schemes for intellectually alert patrons. Uccello had certainly preceded Piero in a fondness for the *mazzocchio* (Fig. 10), but it was Piero's associations with the Lendinara brothers and his manifest influence at Urbino, which promoted the *mazzocchio* to such an extent that it became, together with the armillary sphere and Platonic solids, a cipher of geometrical knowledge and a recurrent challenge to generations of perspectivists, particularly those who specialized in *intarsia* decorations.[2]

[1] M. Daly Davis, *Piero della Francesca's Mathematical Treatises. The 'Trattato d'abaco' and 'Libellus de quinque corporibus regularibus'* (Ravenna, 1977).

[2] F. Arcangeli, *Tarsie* (2nd edn., Rome, 1943); A. Chastel, 'Marqueterie et perspective au XV siècle', *Revue des Arts*, iii (1953), 141–54; B. Ciati, 'Cultura e

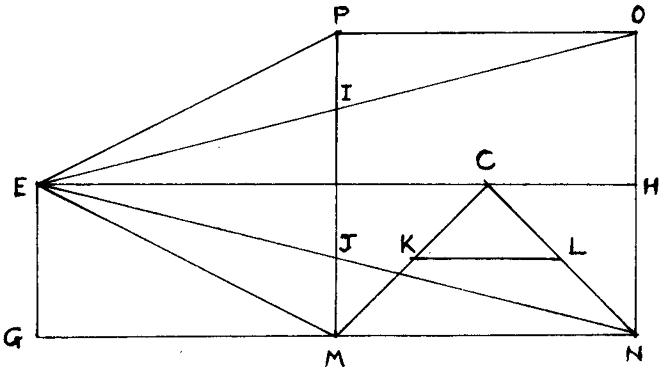


FIG. 9. Piero della Francesca, *Perspective Construction* (M.K.): E, eye; GMN, ground plane; PM, picture plane; MNOP, square to be foreshortened; IJ, intersection of NO on picture plane; H, horizon; C, point to which opposite sides of square recede; KL, perspective projection of NO (horizontal to J and equal to IJ).

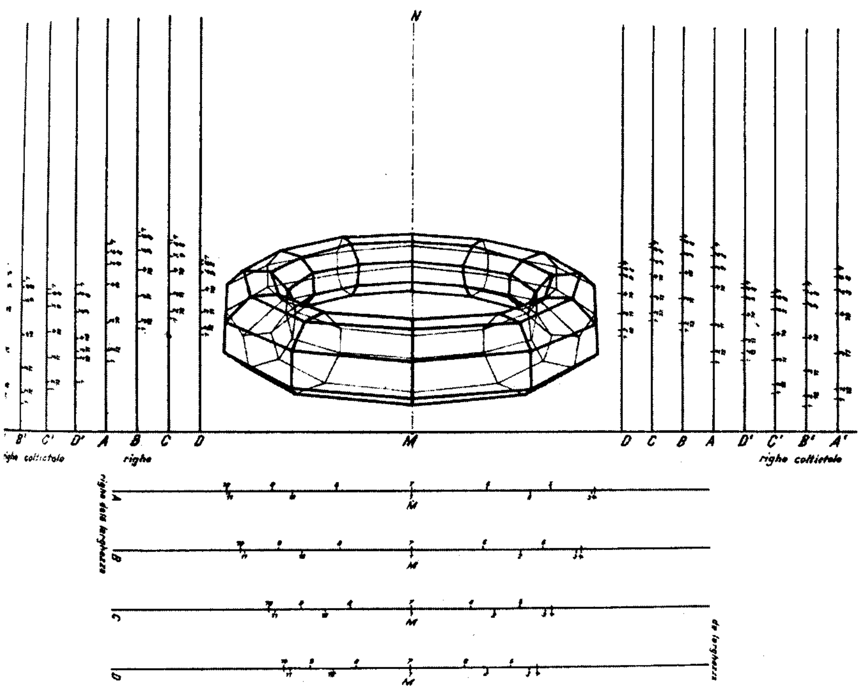


FIG. 10. Piero della Francesca, *Perspective Projection of Mazzocchio*, from *De prospettiva pingendi* (Milan, Biblioteca Ambrosiana) ii, 46.

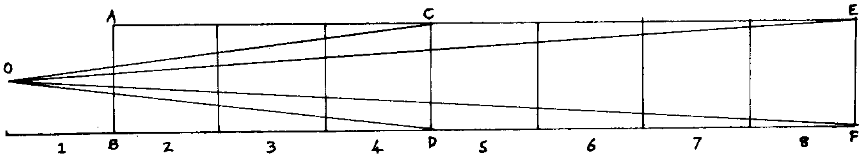


FIG. 11. Leonardo da Vinci, *Proportional Diminution at Intersection* (M.K., based on MS A 8v, Paris, Institut de France): O, eye (1 *braccio* from AB); AB, intersection; CD, 1 *braccio* line (4 *braccia* from O); EF, 1 *braccio* line (8 *braccia* from O).

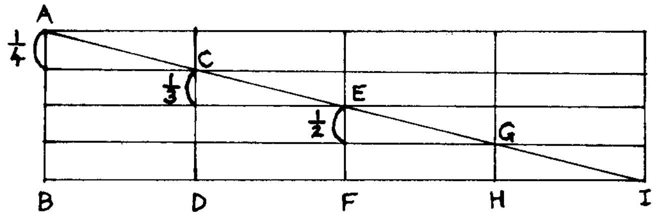


FIG. 12. Leonardo da Vinci, *Geometrical Scale for Proportional Diminution* (M.K., based on Codice atlantico, 132v). IH is equivalent to the distance of the eye to the picture plane. HF is equivalent to the distance of the object to the picture plane. When IH = HF, an object will be diminished by ½ at the intersection. When the distance of the object to the picture plane is doubled (at CD), the object will be diminished to ⅓ of its original size. When the distance is trebled (at AB), the object will be diminished to ¼ of its original size, and so on. The ratios of the actual intersections will be ½ : ⅓ : ¼, etc.

There is no question that Leonardo was greatly attracted by the proportional delights and quantification of Piero's methods, particularly during the earlier phases of his activity as a theorist. He explained (Fig. 11) that 'if you place the intersection one *braccio* from the eye, the first object being at a distance of four *braccia* from your eye will diminish by three-quarters of its height on the intersection; and if it is 8 *braccia* from the eye, by [a further] seven-eighths, and if it is sixteen *braccia* away it will diminish by [a further] fifteen-sixteenths of its height and so on by degrees. As the space doubles, so the diminution will double.'[1] He also provided an abstract 'scale' for indicating the ratios of diminution (Fig. 12), thus an object 1 *braccio* behind the plane will diminish by one-half;

società nel secondo quatrocento attraverso l'opera ad intarsio di Lorenzo e Cristoforo da Lendinara' in M. Dalai Emiliani (ed.), *La Prospettiva rinascimentale* (Florence, 1980), pp. 201–14; and M. Daly Davis, 'Carpaccio and the Perspective of Regular Bodies', in ibid., pp. 183–97.

[1] MS A 8v (Paris, Institut de France), in *The Literary Works of Leonardo da Vinci* (ed. J. P. Richter, 3rd edn., 2 vols., London, 1970), para. 100. Richter's translation looses the sense of Leonardo's statement.

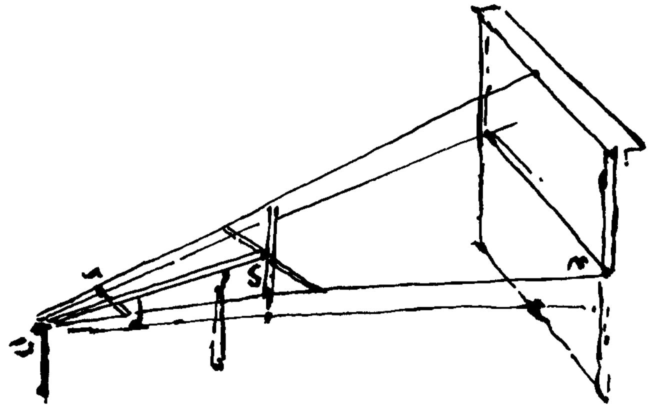


FIG. 13. Leonardo da Vinci, *The* 'Bacolo' *of Euclid* (M.K., based on Codice atlantico, 148^{v}*b*).

when 2 *braccia* behind it will diminish to one-third of its original size; when 3 *braccia* behind it will diminish to one-quarter of its size and so on. Not surprisingly, Leonardo felt that he was dealing with a form of visual music, which he expressed most tellingly in the *Last Supper*, where the ratios of the widths of the tapestries on the wall plane are $1:\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$.[1]

In illustrating Luca Pacioli's *De divina proportione* in 1498 (Pl. II*b*), he was also following in Piero's footsteps, and it is not surprising to find him indulging in perspectival exercises (Pl. II*a*) —perhaps we should say optical gymnastics—of an approved kind. However, Leonardo's fervent adherence to an empiricist tradition lead him to seek the physical causes of geometrical harmonies in a manner not really attempted by Piero. It is symptomatic that his alliegance to proportional triangles was expressed in practical, physical form, in his use of a surveying device which he called the 'bacolo' (stick) of Euclid (Fig. 13). In this search for physical understanding, he turned to medieval optics. However, he found, as many have done since, that attention to the physical mechanisms of sensation and perception provided nothing but a series of complications which resisted neat encapsulation.[2] Medieval optics introduced him to the problem of 'how the eye

[1] Codice Atlantico, 132^{v} (Milan, Biblioteca Ambrosiana), Richter 104; MS BN 2038, 23^{r} (Paris, Institut de France), Richter 102; T. Brachert, 'A Musical Canon of Proportions in Leonardo's *Last Supper*', *Art Bulletin*, liii (1971), 461–6; and M. Kemp, *Leonardo da Vinci. The Marvellous Works of Nature and Man* (London and Cambridge, Mass., 1981), p. 198, fig. 47.

[2] M. Kemp, 'Leonardo and the Visual Pyramid', *Journal Warburg and Courtauld Institutes*, xl (1977), 128–49.

sees' as a cause of perceived effects—the cause of illusions, distortions, and uncertainties, in a way not entertained by Alberti and Piero.

In his late thought, this realization of the complexities of perception was coupled with an increasing sense of the limitations of the single, static viewpoint in representing form—hence the multiviewpoint studies which provide such a brilliant episode in his anatomical illustrations.[1] I should emphasize, however, that his awareness of such complications did nothing in his late thought to disrupt his total commitment to the inherently mathematical operation of nature in all its wholes and its parts. His sense of the interplay between Euclidian geometry and Aristotelian dynamics in determining the physical design of natural form became ever more confirmed in his mind, even if the tasks of the investigator seemed increasingly daunting.[2]

Leonardo's work represents the historical high-point of the creative interaction between the practice of art and advanced research into the geometry of nature—a high-point he shared with his German contemporary, Albrecht Dürer. The sharing is not only a matter of a natural conjunction of interests, but is also a question of Dürer's close contacts with the Leonardo circle. One contact particularly relevant to our present purpose is the arrival of Galeazzo Sanseverino to stay with Dürer's friend, Pirkheimer, in 1499. Galeazzo had been the dedicatee of the supreme manuscript copy of Luca Pacioli's *De divina proportione*.[3]

That Dürer shared Piero's and Leonardo's interests in the proportionality of perspective is underlined by his diagram of geometrical ratios (Fig. 14), which is closely related to those we have already encountered. Like Leonardo, he was also interested in such problems as mean proportionals and the transformation of one geometrical form into another without loss of area or volume. For example, he provided four solutions to the 'Delic' problem of doubling the volume of a given cube, a problem that involves geometrical ratios.[4] The two methods of perspective and shadow projection in his treatise follow immediately and naturally from such

[1] M. Kemp, 'Dissection and Divinity in Leonardo's Late Anatomies', ibid., xxxv (1972), 208–9.

[2] Kemp, *The Marvellous Works . . .*, pp. 293 ff.

[3] *Albrecht Dürer. The Painter's Manual* (ed. W. L. Strauss, New York, 1977), p. 31.

[4] A. Dürer, *Underweysung der Messung* (Nuremberg, 1525), trs. by J. Camerarius as *Institutiones Geometricae* (Nuremberg, 1532), in Strauss (ed.), op. cit., pp. 346–57.

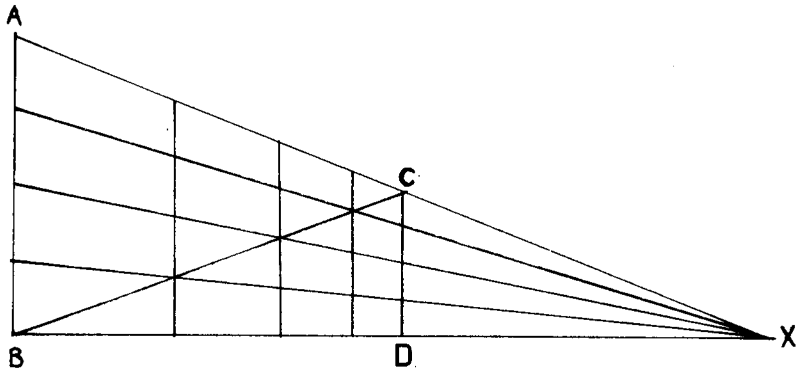


FIG. 14. Albrecht Dürer, *Proportional Diagram* (M.K., based on *Underweysung der Messung* (Nuremberg, 1538), i, fig. 51). Lines of different length, AB and CD are perpendicular to BX. AC is extended to X. AB is divided into equal parts and joined to X. The diagonal BC is drawn. Perpendiculars are drawn through the intersections. The perpendiculars are in a proportional series.

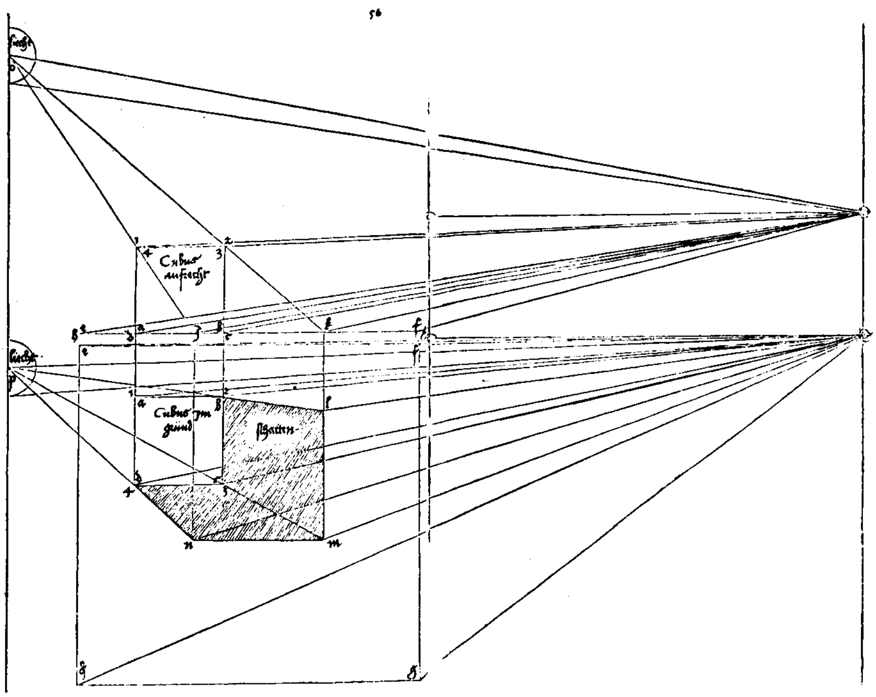


FIG. 15. Albrecht Dürer, *Perspectival Projection using Elevation and Plan of Cube with Accompanying Shadow*, from *Underweysung der Messung* (Nuremberg, 1525), iv, fig. 56.

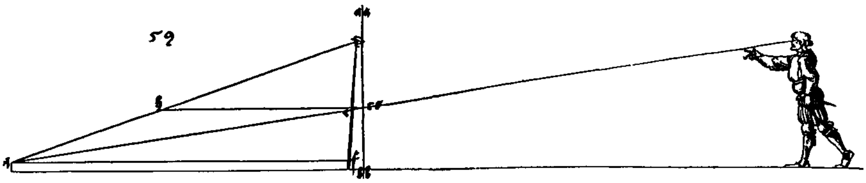


FIG. 16. Albrecht Dürer, *The 'Shorter Way' of Perspective*, from *Underweysung der Messung*, iv, fig. 59. (Note: a line should also be drawn from the base of the square at f to the eye. The resulting intersection on the picture plane (a–a/b–b) would provide the proper horizontal location for the base of the square in perspective projection.)

propositions of proportional geometry. His first method (Fig. 15) involved the full scale procedure of transformation by intersection, using plan and elevation in a manner akin to Piero.

His second method, his so-called 'shorter way' (Fig. 16), was his own variation on the Albertian scheme, but one which introduced an unexpected error, in that the positioning of the intersecting plane away from the foot of the square to be foreshortened is not taken into account at the base of the intersection.[1]

Dürer's most positive contribution, however, was not as an innovator in the theory of pictorial perspective but in the cognate matter of conic sections. Taking up the classical challenge of Apollonius, he provided an entirely logical and accurate method of producing an oblique section of a cone (Fig. 17).[2] His procedure exploited the techniques of transformation which he had learnt in his studies of the intersection of the 'visual pyramid' as the central element in artistic perspective. His solution won a fair measure of mathematical recognition. And we will not be altogether surprised to find that a later perspectivist, Desargues, was also to make a significant contribution to the study of conic sections.

Within the work of Alberti, Piero, Leonardo, and Dürer, the Euclidian components were sufficiently prominent to be readily compatible with the major phase of Euclidian scholarship during the sixteenth century. The middle years of the century were marked by a series of important editions and translations of Euclid's *Elements* and *Optics*. In 1543 Niccolò Tartaglia provided an influential Italian translation of the *Elements*, in which he acknowledged perspective as one of those disciplines which

[1] Ivins, *On the Rationalisation of Sight*, pp. 36–9.

[2] Although Dürer's method presents practical problems, it is sound in theory. E. Panofsky, *The Life and Art of Albrecht Dürer* (4th edn., Princeton, 1955), p. 255, overstates the 'problems' in Dürer's construction.

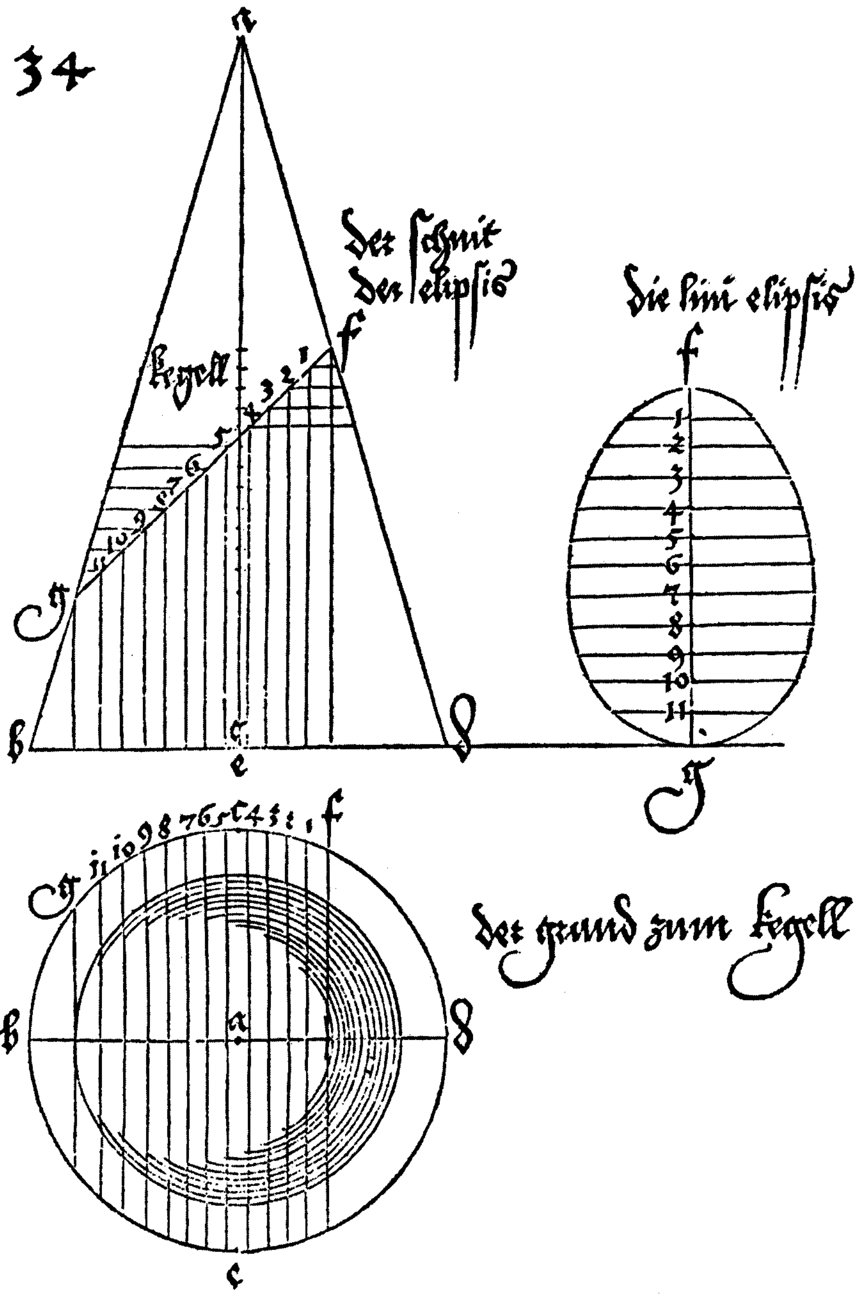


FIG. 17. Albrecht Dürer, *Method for Determining Oblique Section of Cone*, from *Underweysung der Messung*, i, fig. 34.

'mediate between mathematics and natural science'.[1] His own fundamental treatise on ballistics begins with an illustration (Pl. III) of Euclid opening the door to the secure realm of the mathematicizing disciplines, while Plato brandishes his motto—'let noone who is destitute of geometry, enter here'—at the entrance to the inner sanctum.[2] Thus Tartaglia could write with reference to the science of fortification, that 'forma' was more important than 'materia'.[3]

Each of the major contributors to perspective science around the middle of the century were involved in the Euclid revival. Federigo Commandino published his translation of Euclid in 1575.[4] Ignazio Danti's translation of Euclid's *Optics* was published in 1573, ten years before his commentary on Vignola's perspective theories.[5] Daniele Barbaro credited his knowledge of perspective to Giovanni Zamberto, whose brother Bartolomeo was a pioneer translator of Euclid from the Greek, and he aided the publication of Zamberto's 1537 editions of the *Elements* and *Optics*.[6] Symptomatic of this Euclidian climate was an artistic dispute in which Martino Bassi wished to attack Tibaldi in Milan for not following the proper perspectival principles in the background of a relief to be sited high in the cathedral. Bassi's most damning indictment was that the artist was displaying his ignorance of Euclid.[7] This is the Euclidian context in which the mathematical specialization and 'abstraction' of perspective science began to take place.

Barbaro, patriarch elect of Aquileia, editor of Vitruvius and noble patron of the arts, for his part attempted to create a comprehensive, balanced review of the geometrical, physical, and

[1] *Euclide Megarense Philosopho* (trs. and ed. N. Tartaglia, Venice, 1543), preface. For Tartaglia see P. L. Rose's richly detailed account of humanist mathematics, *The Italian Renaissance of Mathematics* (Geneva, 1975), pp. 151–3.

[2] N. Tartaglia, *La nova scientia* (Venice, 1550), frontispiece. Leonardo paraphrased Plato's motto in connection with his anatomical studies (Kemp, *The Marvellous Works* . . ., p. 293).

[3] J. Hale, 'To Fortify or not to Fortify? Machiavelli's Contribution to a Renaissance Debate', *Renaissance War Studies* (London, 1983), p. 194, quoting Tartaglia's *Quesiti e inventioni* (Venice, 1546), 69^{r-v}.

[4] *Euclidis elementorum libri XVI conversi* (trs. and ed. F. Commandino, Pesaro, 1572), and *Degli Elementi d'Euclide* (trs. and ed. F. Commandino, Pisa, 1575).

[5] *La Prospettiva di Euclide* (trs. and ed. I. Danti, Florence, 1573).

[6] M. Daly Davis, *Piero* . . ., p. 6, and 'Carpaccio . . .', note 10 (where Bartolomeo is given as 'Daniele'). For Zamberti see Rose, op. cit., pp. 50–2.

[7] M. Bassi, *Dispareri in materia d'archittetura* . . . (Brescia, 1572) in *Scritti d'arte del Cinquecento*, ed. P. Barrochi (Milan and Naples, 1973), pp. 1799–832; see E. Panofsky, *La Prospettiva come forma simbolica* (ed. G. Neri with essay by M. Dalai, 3rd edn., Milan, 1976), pp. 108–9.

physiological aspects of perspective. From geometry the subject was said to gain its 'reason'; from physics came an understanding of the nature of vision.[1] His geometry was firmly Euclidian, and his optical science was an updated version of medieval visual theory in the Pecham mode. The major thrust of his argument, however, was weighted heavily on the geometrical side of the equation, and he drew extensively upon Piero della Francesca, in spite of his damning opinion that Piero wrote for 'idiots'.[2] His variations upon the *mazzocchio* theme (Fig. 18) stress this relationship. Like Piero, he criticized contemporary practitioners for ignorance of the force of visual angles: artists 'are content with a basic procedure [*semplice pratica*]', which does no justice to the divine authority of mathematics.[3] The emphasis in his *La Pratica della perspettiva*, published in 1569, is upon perspective as an integral facet of the 'secret art' of the world and the cosmos. The geometrical solids, upon which he concentrated in book III, were particularly revered, in that they 'signified for Plato the elements of the world, and heaven itself, and by the secret intelligence of their forms we ascend to the loftiest speculations'.[4] He sang an appropriate hymn in praise of the 'proportional reason of time and distance', linking perspective with astronomy, horology, and cartography in his treatise.[5] His use of the *camera obscura* phenomenon, of which he provides an early and authoritative account, accurately reflects the tenor of his writings. The 'photographic' image produced in the *camera obscura*, which he regards as 'a most beautiful experience', is valued above all as a means of 'teaching us the proportional diminution of objects, helping us in every way to formulate the precepts of the art'.[6]

Barbaro's unpublished manuscript treatise on perspective, which follows Piero and Pacioli in concentrating upon the depiction of Platonic solids, is characteristic of his approach, and reflects the way in which such bodies had become objects of aesthetic-cum-cosmological contemplation. Pacioli, as his portrait shows and as is documented, constructed the bodies in crystal and other materials, providing one set for the edification of the Florentine

[1] D. Barbaro, *La Pratica della perspettiva* (Venice, 1569), I, paras. 5–7, p. 7.

[2] Ibid., proemio.

[3] Loc. cit.

[4] Ibid., II, para. 8, p. 37.

[5] D. Barbaro, *I dieci libri dell'Archittetura di Vitruvio* (Venice, 1569), p. 97 (intro. to book III).

[6] Barbaro, *Prat.*, IX, para. 5, pp. 192–3.

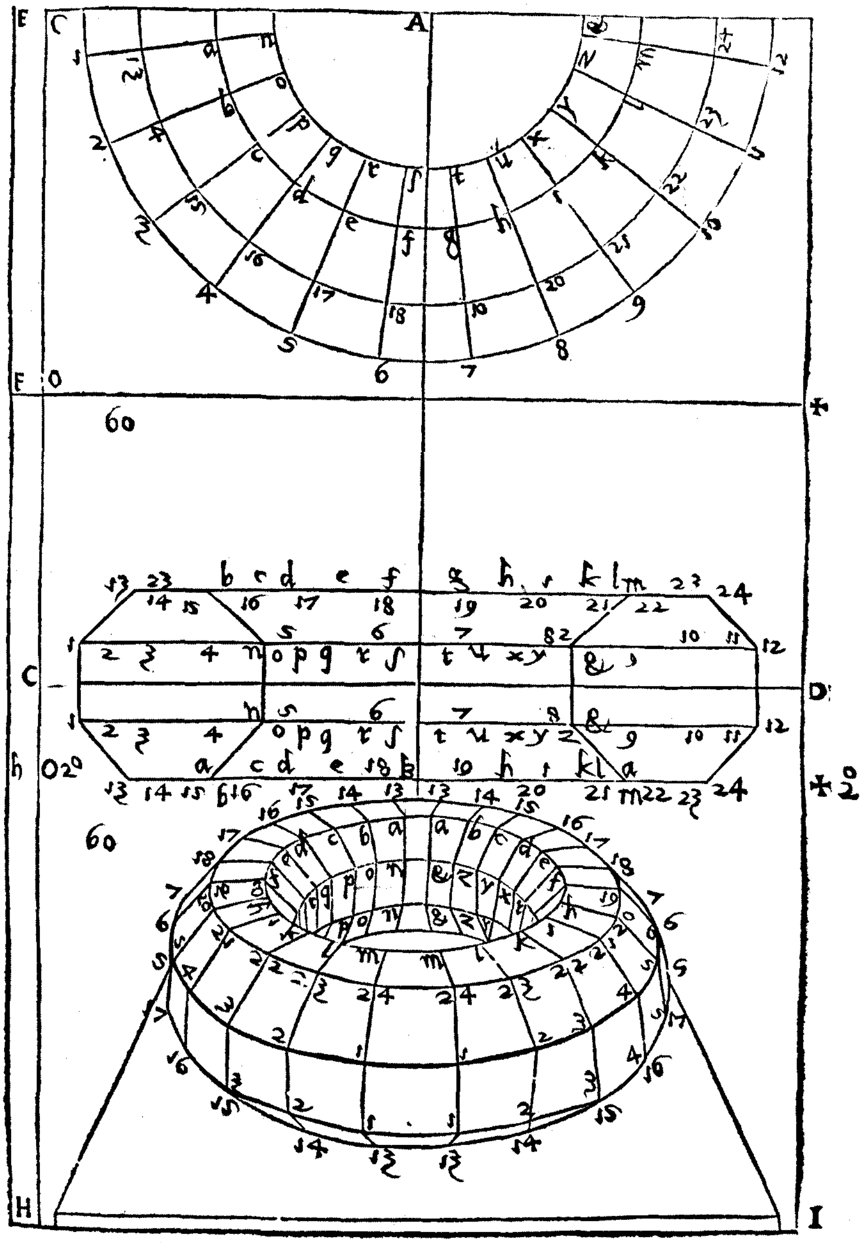


FIG. 18. Daniele Barbaro, *Perspective Projection of Mazzocchio*, from *La pratica della perspettiva* (Venice, 1569).

council.[1] In 1561, the Nuremberg mathematician, calligrapher and first biographer of northern artists, Johannes Neudorfer, was portrayed by Neufchatel initiating his son into the mysteries of a dodecahedron, using a hollow model of the type invented by Pacioli and illustrated by Leonardo.[2] Not infrequently the depiction of such bodies, particularly in their stellated forms (Pls. IV, V*b*), developed an overtly decorative quality. Nowhere was this more so than in the North, where Cousin, Jamnitzer (Pl. V*a*), Stoer, and Lencker worked mannerist variations upon these intellectual themes in an 'applied arts' manner, without necessarily explaining the full supporting apparatus of projective geometry.[3] Jamnitzer, who devised 150 variations, came like Lencker from a background in goldsmithing. In Italy, Jamnitzer-style solids were used as cosmological ciphers in the paintings in the Sala Clementina of the Vatican by the Alberti brothers, who came appropriately enough from Piero della Francesca's Borgo San Sepolchro.[4]

Cherubino Alberti, in his guise as an engraver, had also contributed illustrations to the other great perspective treatise of the period which attempted a synthesis of mathematical, optical, and pictorial knowledge—Ignazio Danti's commentary on Vignola's *Le due regole della prospectiva pratica* published in 1583.[5] Vignola had been a practising painter who became an architect of real quality and geometrical skill, while Danti was a distinguished member of a notable Perugian family of artists and authors spanning three generations.[6] Danti, a Dominican priest, was Papal cosmologer, cartographer, and an avid student of classical mathematics.

[1] For a good exposition of the portrait see M. Daly Davis, *Piero . . .*, pp. 67–80. M. Dalai Emiliani, 'Figure rinascimentali dei poliedri Platonici. Qualche problema di storia e di autografia', *Fra Rinascimento manierismo e realtà* (*in memoria di Anna Maria Brizio*) (Florence, 1984), pp. 7–16.

[2] Munich, Alte Pinakothek; see R. Peltzer, 'Nicolas Neufchatel und seine Nürnberger Bildnisse', *Münchner Jahrbuch der bildenden Kunst*, iii (1926), 187–231. Neüdorfer's biographies are edited by G. Lochner, 'Nachrichten von Kunstlern und Werkleuten', *Quellenschriften für Künstgeschichte*, x (Vienna, 1875).

[3] A useful review of these and other sixteenth-century treatises is provided by L. Vagnetti, 'Il Processo di maturazione di una scienza dell'arte: la teoria prospettica nel Cinquecento' in *La Prospettiva rinascimentale*, pp. 427–74.

[4] K. Herrmann-Fiore, 'Giovanni Albertis Kunst und Wissenschaft der Quadratur eine Allegorie in der Sala Clementina des Vatikan', *Mitteilungen des Kunsthistorischen Institutes in Florenz*, xxii (1978), pp. 61–84.

[5] J. Barozzi (Vignola), *Le due regole della prospettiva pratica con i commentari del R.P.M. Egnatio Danti* (Rome, 1583). See T. Kitao, 'Prejudice in Perspective: A Study of Vignola's Perspective Treatise', *Art Bulletin*, xliv (1962), 173–94.

[6] Ignazio (Egnatio) published his grandfather's work, *La Sfera tradotta da Pier*

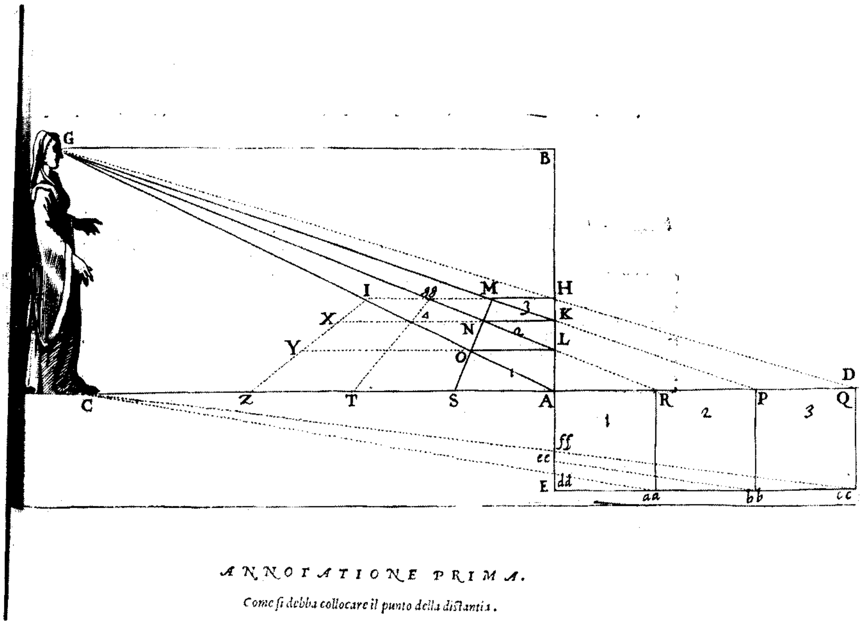


FIG. 19. Jacopo Barozzi da Vignola, *Perspective by Intersection and Distance Point Methods*, from *Le due regole della prospettiva*, Rome, 1583. The intersections produced on the plan (below the line CD), i.e. A ff, A ee, and A dd, equal the dimensions MH, NK, and OL produced by the diagonals from H, K, and L to the distance point, G.

Vignola's concern in his treatise, unpublished in his own lifetime, had been to demonstrate both the Albertian ('intersection') method and the distance-point ('bifocal') technique, showing the basic harmony between them (Fig. 19). This he did diagramatically, though not strictly speaking by geometrical proof. The more abstract demonstrations found practical expression in a series of perspective machines illustrated by Vignola and Danti, including a device invented by a little-known but potentially interesting perspectivist, Tommaso Laureti, one of a group of painters in Bologna who established that city's speciality in illusionistic decorations.[1] We will encounter one of Tommaso's machines later (Fig. 22).

Vincenzo Danti . . . e commentata da Frate Ignazio (Florence, 1571). He edited and translated scientific texts, e.g. *La Sfera di Proclo liceo tradotta da Ignazio Danti . . .* (Florence, 1573), and *Latino Orsini. Trattato del radio latino* (Rome, 1586), as well as producing original treatises of his own, e.g. *Prima volume dell'uso et fabbrica dell'astrolabio e del planisfero* (Florence, 1578). His brother, Vincenzo, was a considerable sculptor and author of the *Trattato delle perfetti proporzioni*, of which only *Il primo libro del trattato . . .* (Florence, 1567) survives.

[1] For Laureti and his fellow illusionists in Bologna and their links with Danti see E. Sjöström, *Quadratura: Studies in Italian Ceiling Painting* (Stockholm, 1978).

Danti's commentary, which outweighed Vignola's text, brought a full knowledge of geometry and the most advanced optics to bear upon Vignola's procedures. He had been greatly helped in this by Risner's great double publication in 1572 of Alhazen's and Witelo's optical treatises.[1] Danti's efforts to bring the Euclidian, Arabian, and pictorial strands of perspective into harmony are probably as successful as they could be, given the inherent problems, but there are none the less clear tensions between the artist's *pratica* of Vignola's original, and the complex *scientia* of Danti's sections. Danti was all too aware that such complexities as the rotation of the eye provide limitations for the artist's theory.

Danti's work, to a greater degree than Barbaro's, signifies the increasing absorption of perspective into professional mathematics and the highly technical world of the mathematical sciences. Even more uncompromising was Federigo Commandino's annexing of artistic perspective in his complex work on techniques of Ptolemaic projection, his *Commentary on the Planisphere* . . . of 1558.[2] Appropriately, Commandino was a native of Urbino, the first home of Piero's treatises, and his book was dedicated to Cardinal Ranuccio Farnese, a member of the family who granted extensive patronage not only to Commandino but also to Vignola. The context of his perspective demonstrations (Fig. 20) was his discussion of stereographic projection in the Ptolemaic tradition, distinguishing stereographic and orthographic projection in the mapping of the heavens and earth on flat surfaces. Such Ptolemaic concerns may have already impinged upon artistic perspective in the fifteenth century. Ptolemaic spheres had come to rival *mazzocchii* and the Platonic solids as set-pieces of the perspectivist's art—appearing *inter alia* in the Urbino *intarsie*, and the St Augustines by Botticelli and Carpaccio.[3] Furthermore, as we will see, the techniques for projecting a planisphere became closely entangled with the artist's science of shadow projection.[4] I am not, however, inclined to believe the hypothesis that the 'rediscovery' of Ptolemy played a

[1] Witelo, *Opticae thesaurus Alhazeni* . . . (ed. F. Risner, Basel, 1572).

[2] F. Commandino, *Ptolomaei Planisphaerium* . . . (Venice, 1558). See R. Sinisgalli, 'Gli studi di Federigo Commandino sul Planisfero Tolemaico come elemento di rottura nella tradizione della teoria prospettica della Rinascenza' in *La Prospettiva rinascimentale*, pp. 475–86; and Rose, op. cit., pp. 185–214.

[3] The *studiolo* of Federigo da Montefeltro, Palazzo Ducale, Urbino; Botticelli, *The Vision of St. Augustine*, Florence, Ognissanti (refectory); Carpaccio, *The Vision of St. Augustine*, Venice, Scuola di S. Giorgio degli Schiavoni.

[4] T. da Costa Kauffman, 'The Perspective of Shadows: the History of the Theory of Shadow Projection', *Journal Warburg and Courtauld Institutes*, xxxviii (1975), 258–87.

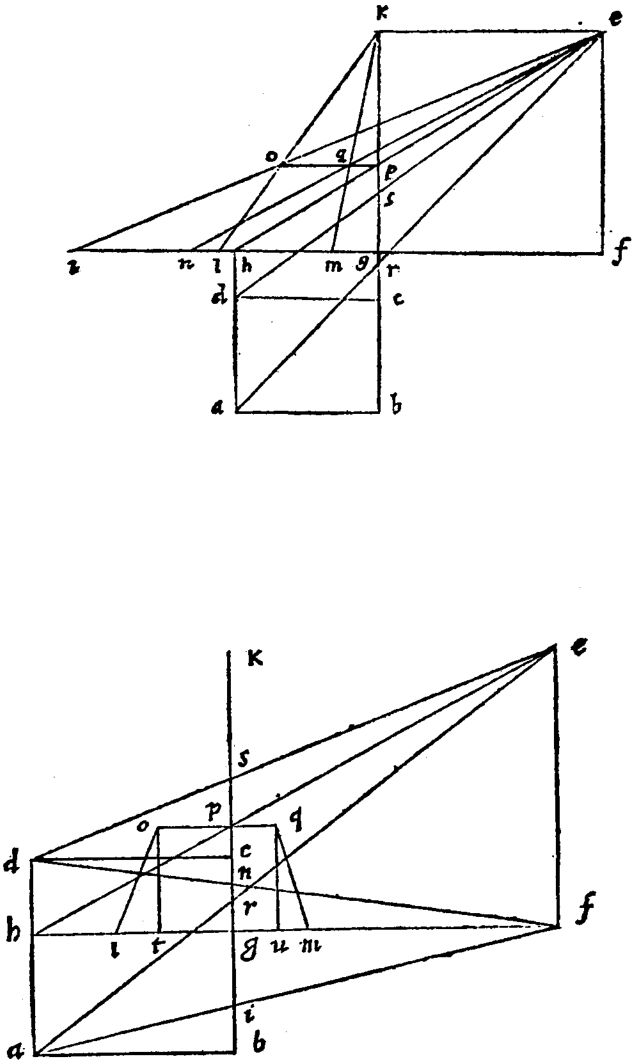


FIG. 20. Federico Commandino, *Two Methods of Perspectival Projection of Rectangle*, (from *Ptolomaei Planisphaerium* . . . (Venice, 1558): e, eye; abcd, square to be foreshortened; f, location of viewer on ground plane. *In upper diagram*, a, b, c, and d are transferred to i, l, m, and n respectively; i, n, and h are joined to e; l and m are joined to k; pqo is drawn parallel to if. The desired projection is oqml. *In lower diagram* c and b are transferred to m and l respectively; h is joined to e; a and d are joined to f; i and n are transferred to u and t. Verticals are raised from u and t to meet the horizontal through p. The desired projection is oqml.

genuinely formative role in Brunelleschi's and Alberti's pioneering efforts.[1]

Commandino's actual demonstrations, in longer and abbreviated versions, work variations on the technique of 'transformation by intersections' in the Piero manner, but their uncompromisingly flat, abstract, geometrical presentation makes no concession to the three-dimensionality of actual objects—a sense of three-dimensionality retained by both Barbaro and Vignola.

It was this uncompromisingly geometrical vision which passed to his greatest pupil in Urbino, the Pesaro nobleman, Marchese Guidobaldo del Monte. Guidobaldo's *Perspectivae libri sex*, published in 1600, represents a crucial step in the transformation of perspective into the science of projective geometry.[2] It is arguably the most systematic, comprehensive, and intellectually cogent book on perspective ever published, progressing methodically from the projection of points—for which he provides twenty-three different techniques in book II—through geometrical solids and cast shadows, to scenography in book VI. Guidobaldo finally achieved the geometrical proof of the *punctum concursus*, the 'point of convergence', not only for the Albertian parallels perpendicular to the intersection, but for any given group of parallels in any plane. Guidobaldo demonstrated that the point at which a line through the eye parallel to a given line behind the intersecting plane meets that plane is the common vanishing point for all other lines parallel to the given line in any plane behind the intersection.[3]

The steps by which he achieved his formulation make clear reference to the Vignola-Danti treatise, and one of his demonstrations (Fig. 21) resembles the layout of Tommaso Laureti's

[1] For discussions of Ptolemy and early perspective see S. Y. Edgerton jnr., *The Renaissance Rediscovery of Linear Perspective* (New York, 1975); M. Kemp, 'Science, Non-Science and Nonsense: the Interpretation of Brunelleschi's Perspective', *Art History*, i (1978), 147-8; and K. Veltman, 'Ptolemy and the Origins of Linear Perspective', in *La Prospettiva rinascimentale*, pp. 403-7.

[2] Guidobaldo del Monte, *Perspectivae libri sex* (Pesaro, 1600). For Guidobaldo's mathematics and mechanics see Rose, op. cit., pp. 222-36. Highly original anticipations of Guidobaldo's ideas (and those of Stevin) appeared in Giovanni Battista Benedetti's *De rationibus operationum perspectivae* in *Diversarum speculationum mathematicarum et physicarum liber* (Turin, 1585). However, Benedetti's treatise appears to have exercised little direct influence and stood largely outside the particular line of descent I am tracing. See J. V. Field's fine exposition in a forthcoming article, 'Giovanni Battista Benedetti on the Mathematics of Linear Perspective', *Journal Warburg and Courtauld Institutes*, which became available to me after the present paper was completed.

[3] B. Carter, 'Perspective', *Oxford Companion to Art* (ed. H. Osborne, Oxford, 1970), p. 850.

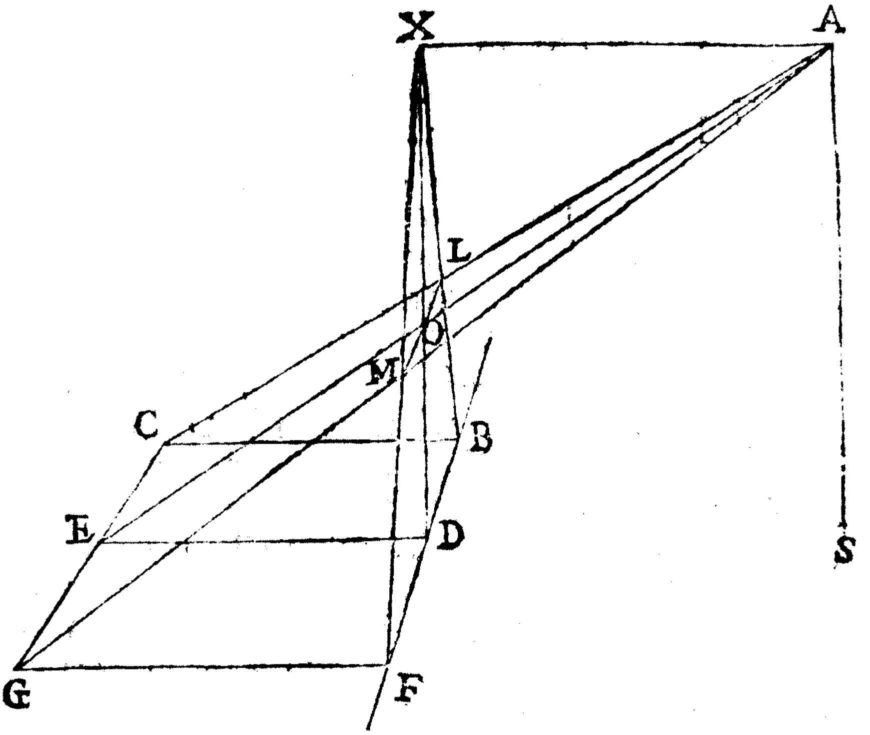


FIG. 21. Guidobaldo del Monte, *The 'Punctum Concursus' in the Projection of Parallel Lines*, from *Perspectivae libri sex* (Pesaro, 1600): AS, viewing height; BDF, base of plane on to which points CEG are to be projected. CE and G are joined by 'visual rays' to A. LOM will be the points in projection. FM, DO, and BL are extended to X, the 'punctum concursus'.

perspective machine, as convincingly illustrated (Fig. 22) and subjected to geometrical analysis by Danti. The ultimate goal of his formulation, however, was rigorously concerned with the accomplishing of the projective construction within a single plane. His definitive demonstrations rotate the picture plane into the ground plane (Fig. 23). Lines parallel to the outlines of the form to be projected are drawn from the viewing position (S) to the intersection (FB). 'Verticals' equal to the viewing height are raised perpendicularly from the resulting points and are joined to further points on the intersection which are produced by the extended sides of the original figure (G, H, K). The 'tops' of the verticals are joined to the second set of points on the intersection, and these resulting lines inscribe the required figure in accurate projection. Few, if any artists, would be prepared to master this—and they would gain little comfort from Guidobaldo's illustrations of scenography

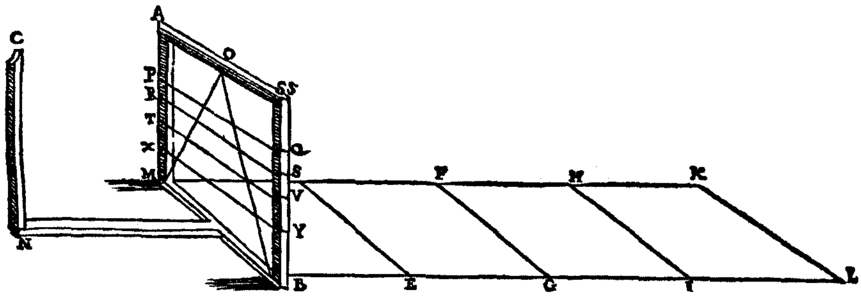


FIG. 22. Jacopo Barozzi da Vignola, *Tommaso Laureti's Perspective Machine*, from *Le due regole della prospettiva* (Rome, 1583).

(Fig. 24), which suggested that all his mathematical brilliance would not itself produce a truly artistic effect.

We may suspect that not a few artists—and most subsequent art historians—would sympathize with Federico Zuccaro, who wrote that 'the art of painting does not derive its principles from the mathematical sciences, nor does it need recourse to them in order to learn rules or methods for its practice or even to give rationality to its theory'.[1] Censuring Leonardo and Dürer for their 'fruitless' endeavours, Zuccaro echoed Michelangelo in preferring to rely upon the artist's innate judgement—what Michelangelo called the 'compasses in the eye'.[2]

For Guidobaldo, as for Danti, Barbaro, and Tartaglia, the ultimate justification of *perspectiva* as the true foundation of art lay in its integral position within the broader realm of all mathematically directed pursuits and in the ultimately mathematical rationale of the cosmos. The interpenetration of pure mathematics and material techniques is nicely symbolized by an instrument called 'Guidobaldo's sector'. Based upon proportional compasses invented by Commandino at the instigation of Bartolomeo Eustachio and in its turn the ancestor of Galileo's *squadra*, it could be used to solve problems of pure mathematics (arithmetic and geometry), or applied to such questions as rates of currency exchange, the apportioning of soldiers in battle formations, the calibres of guns and the surveying of property.[3] Such skills would

[1] F. Zuccaro, *L'Idea dei scultori, pittori e archittetti* (Rome, 1607), II, para. 6 (trs. E. Holt, *A Documentary History of Art*, 2 vols., Princeton, 1958, II, p. 90); also *Scritti d'arte di Federico Zuccaro* (ed. D. Heikamp, Florence, 1961); and Panofsky, *Idea*, p. 75.

[2] D. Summers, *Michelangelo*..., pp. 352 ff., esp. pp. 368–79.

[3] See P. L. Rose, 'The Origins of the Proportional Compass from Mordente to Galileo', *Physis*, x (1968), 53–69; also E. Rosen, 'The Invention of the Reduction Compass', ibid., pp. 306–8; and G. Galilei, *Operations of the Geometrical and Military Compass* (1606) (trs. S. Drake, Washington, 1978).

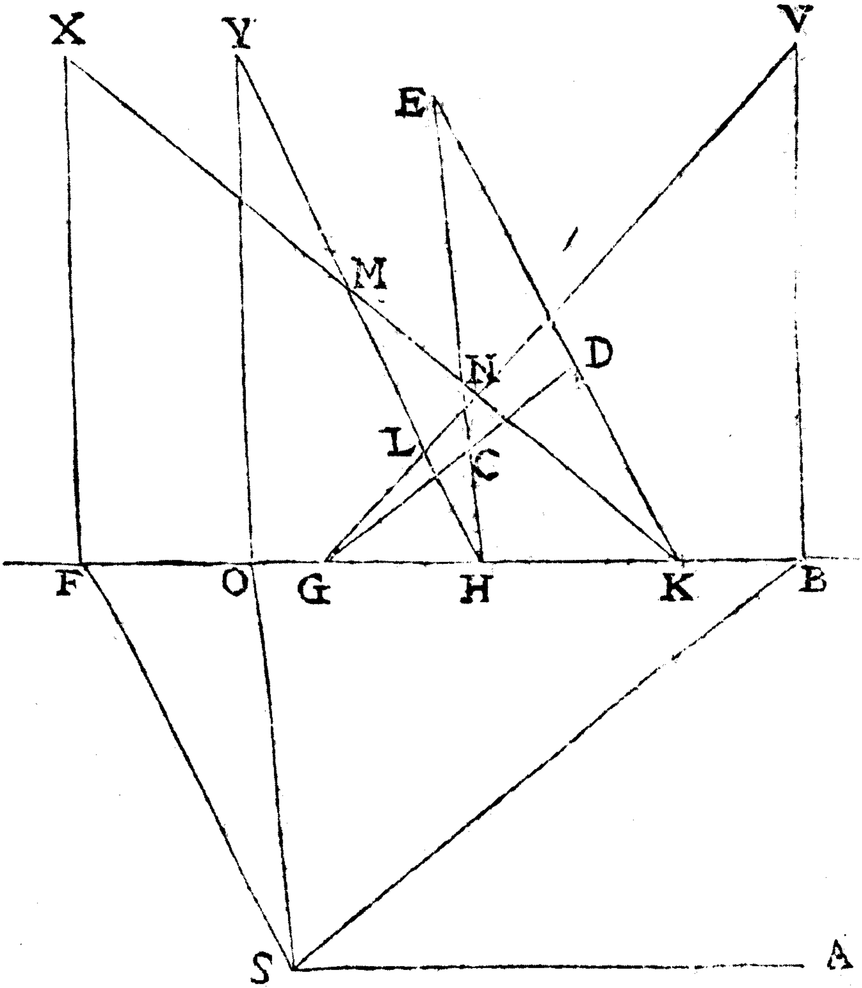


FIG. 23. Guidobaldo del Monte, *Perspective Projection of Triangle*, from *Perspectivae libri sex*: SA, viewing height; FB, base of plane on to which triangle CDE is to be projected; SF, drawn parallel to DE; SO, drawn parallel to CE; SB, drawn parallel to CD. The perpendiculars FX, OY, and BV each equal SA. ED, EC, and DC are extended to K, H, and G respectively. X is joined to K, Y to H, and V to G. LMN is the desired triangle.

have been put to good use in Guidobaldo's duties as Inspector General of Fortifications in Tuscany after 1588.

When the creative centre of perspective science moved North, as it largely did after 1600, we will find the link with military science fortified repeatedly. Simon Stevin and Samuel Marolois, the chief theorists in Holland, both practised as military engineers, as did Girard Desargues in France. The military science of

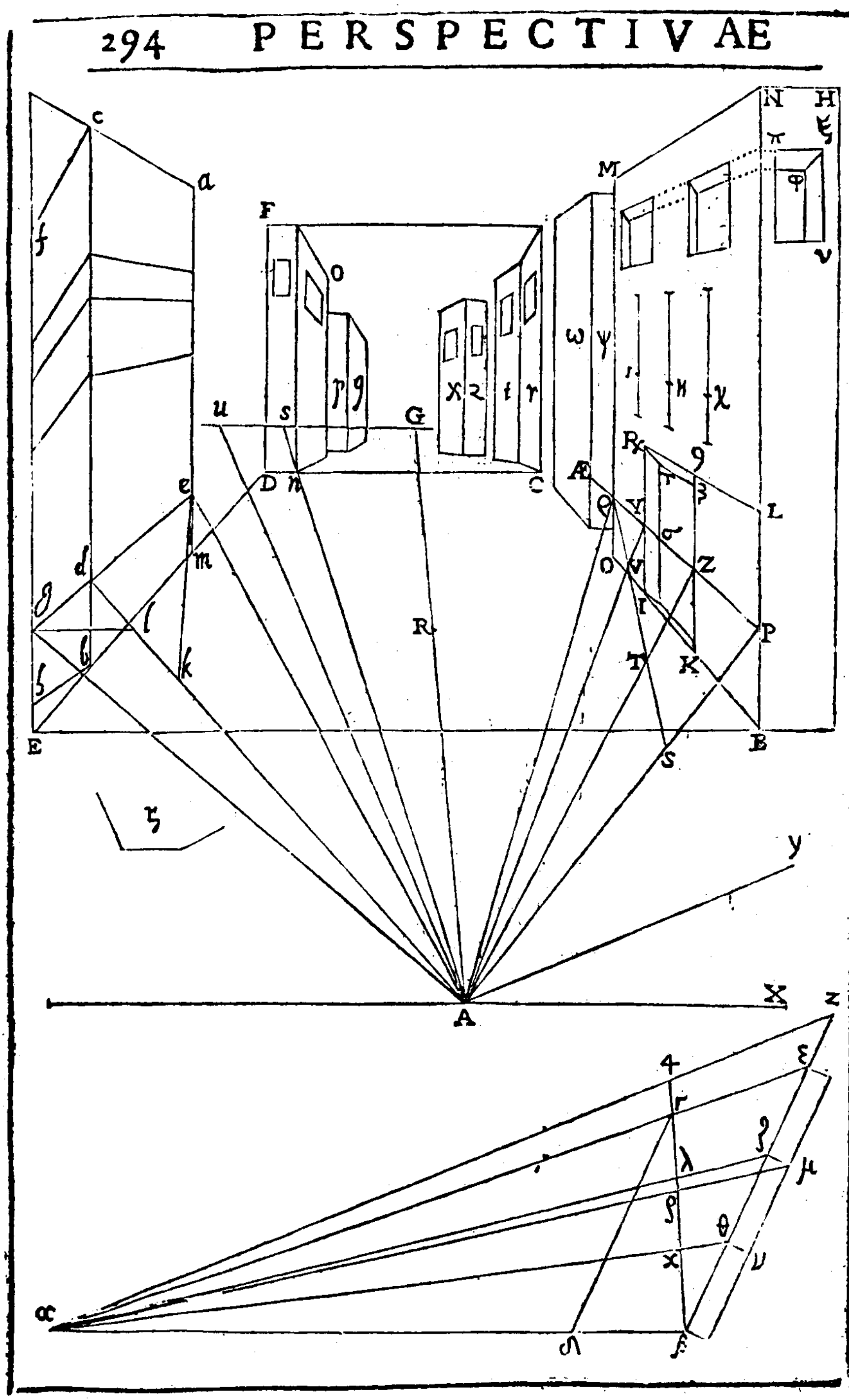


FIG. 24. Guidobaldo del Monte, *Scenographic Perspective*, from *Perspectivae libri sex*.

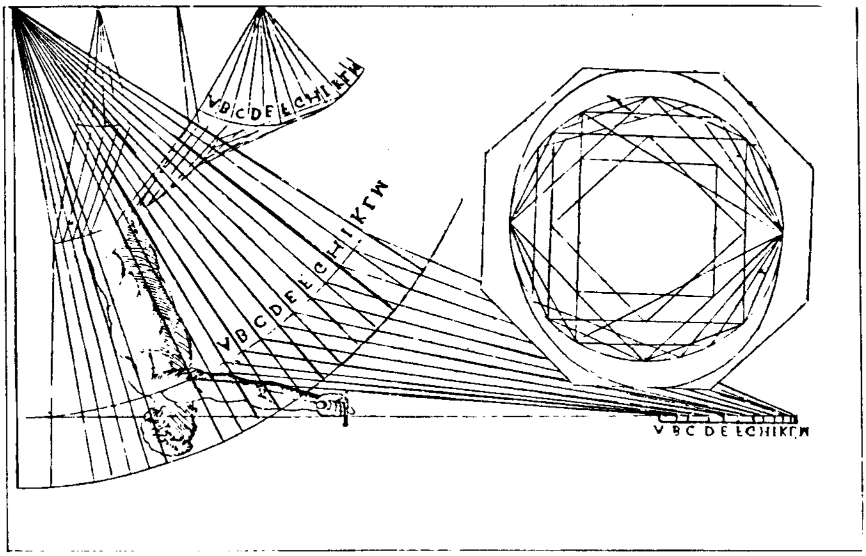


FIG. 25. Carlo Urbino, *Geometrical Diagrams etc.*, from Camillo Agrippa, *Trattato di scientia d'arme* (Rome, 1553).

textbooks had become a battle ground of arithmetical and geometrical theories. As Camillo Agrippa (Fig. 25) wrote, 'this Profession is governed solely by points, lines, intervals, measures and such-like'.[1] All this was to the understandable irritation of not a few of the military men who were responsible for the often chaotic and unfailingly messy business of the actual fighting.[2]

Stevin himself produced an impressively geometrical book on fortifications as a natural extension of his mathematical writings. His book on perspective, *De Sciagraphia* (*Der Deursichtighe*), published in Leiden in 1605, was constructed systematically like a true treatise of Euclidian geometry from definitions, postulates, theorems, propositions, proofs, and corollaries.[3] Like Guidobaldo, he contributed to our understanding of what happens when the

[1] C. Agrippa, *Trattato di scientia d'arme* (Rome, 1553), f. III. See G. Bora, 'La Prospettiva della figura umana. "Gli Scurti" nella teoria e nella pratica pittorica lombarda del Cinquecento' in *La Prospettiva rinascimentale*, p. 312.

[2] J. Hale, *Renaissance Fortification. Art or Engineering?* (London, 1977), p. 35, for Tomaso de Venetia's statement that 'this craft is not to be learned in Bologna, or Perugia or Padua nor out of books, but in action'.

[3] *The Principal Works of Simon Stevin* (ed. E. Crone *et al.*, trs. C. Dikshoorn, Amsterdam, 1955–66), IIb, pp. 798–965. R. Sinisgalli, *Per la storia della prospettiva (1405–1605): Il contributo di Simon Stevin allo sviluppo scientifico della prospettiva artificiale* (Rome, 1978).

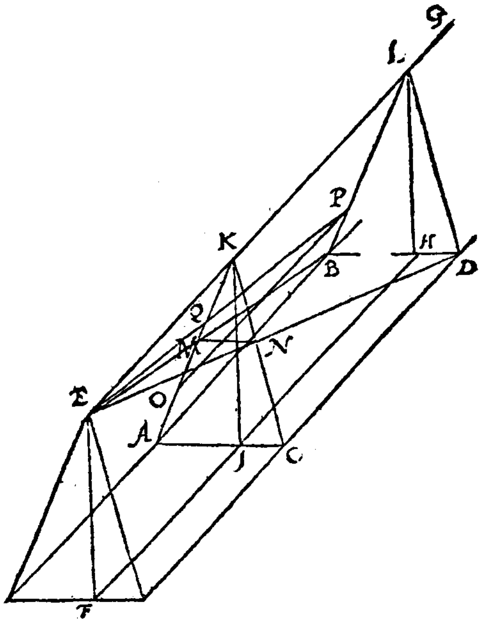


FIG. 26. Simon Stevin, *Demonstration of 'Punctum Concursus'*, from *Der Deursichtighe* (Leiden, 1605): E, eye; ACK, vertical plane (Stevin's 'glass'); BD, points projected to MN.

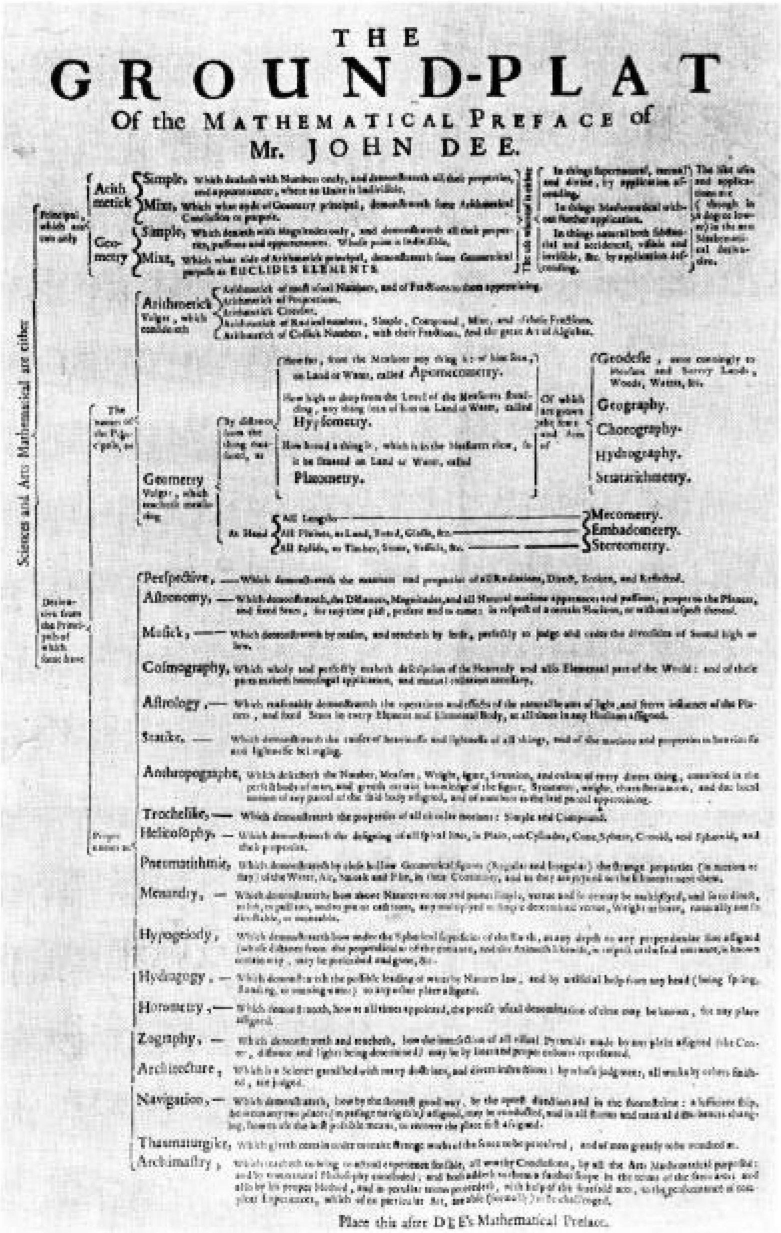
picture plane (or 'glass' as he called it) is folded into the ground plane—formulating an important theorem of homology—and he adopted a series of geometrical demonstrations of the Guidobaldan kind (Figs. 26–7).[1] However, he was also sensibly aware of the artist's problems in relation to his science. He realized that the rotation of the planes was 'seldom required' in practice, though it is needed for 'perfect knowledge'.[2] He also gave the non-specialist reader advice as to how to pick out those practical demonstrations in which perspective is 'set forth mechanically', and he provided suitable 'abridgements', including a straightforward demonstration of the Albertian formula (Fig. 28).[3]

Stevin's fellow mathematician, military expert and surveyor, Samuel Marolois, helped bring Guidobaldan mathematics even closer to the reach of artists in his *Optica sive perspectivae*

[1] *The Principal Works*, p. 791, for the theorem of homology. The inclined picture plane had appeared in Benedetti's *De rationibus* . . .

[2] *The Principal Works*, p. 805.

[3] Ibid., pp. 869 ff.


THE

GROUND-PLAT

Of the MATHEMATICAL PREFACE of

Mr. JOHN DEE.

Sciences and Arts Mathematical are either

Arith metick: Simple, Mixt.

Geo- metry: Simple, Mixt.

EUCLIDES ELEMENTS

Arithmetick Vulgar

Geometry Vulgar

Apomecometry. Hypsometry. Planometry.

Geodesie. Geography. Chorography. Hydrography. Stratarithmetry.

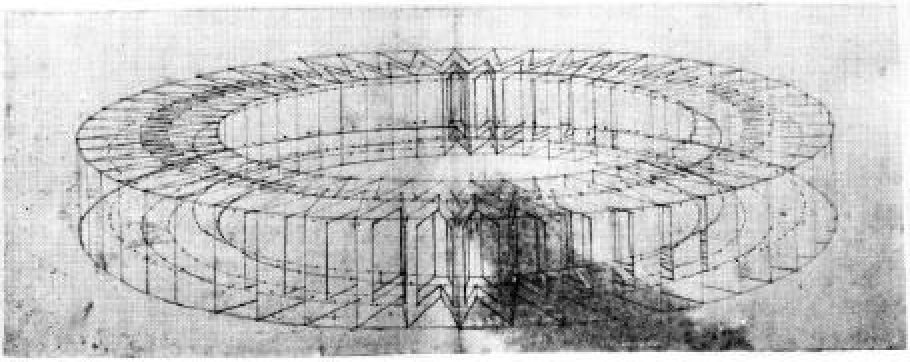
Mecometry. Embadometry. Stereometry.

Perspective, Astronomy, Musick, Cosmography, Astrology, Statike, Anthropographie, Trochelike, Helicosophy, Pneumatithmie, Menadry, Hypogeiody, Hydragogy, Horometry, Zographie, Architecture, Navigation, Thaumaturgike, Archemastry.

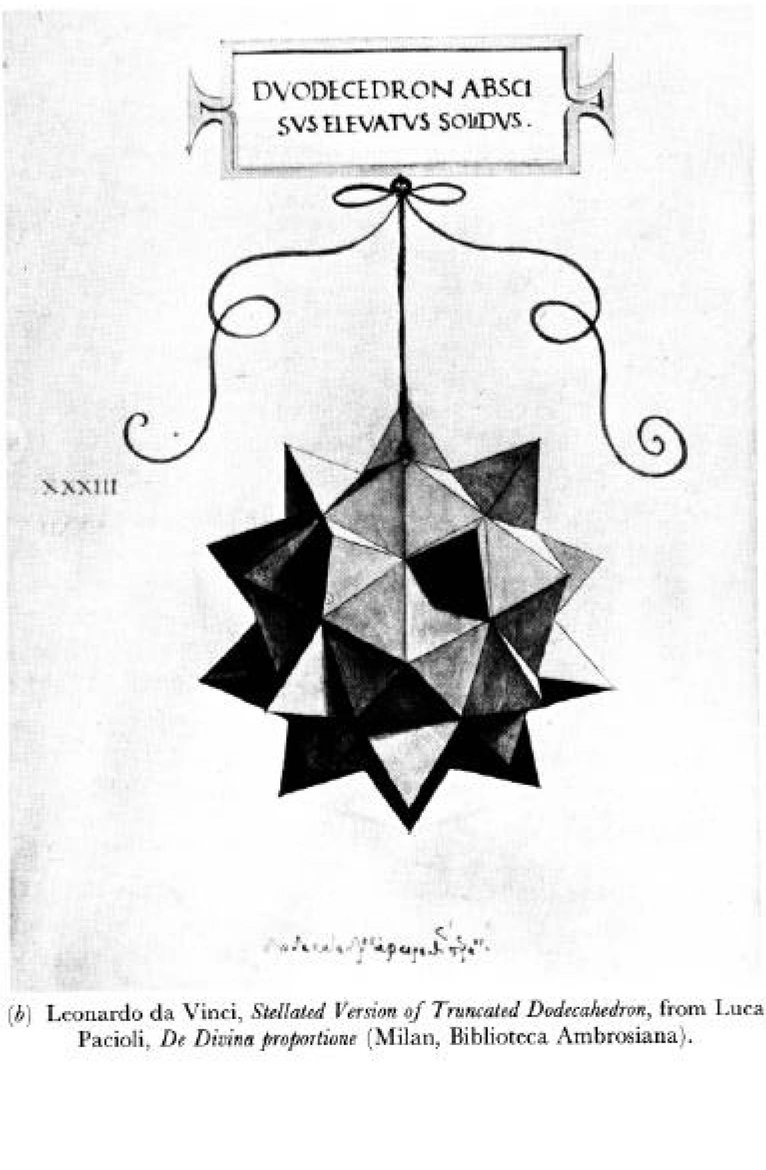
Place this after DEE's Mathematical Preface.

John Dee, '*Ground-Plat*' *of Preface to Euclid*, from J. Leeke and G. Serle (eds.), *Euclid's Elements* (London, 1651).

PLATE II



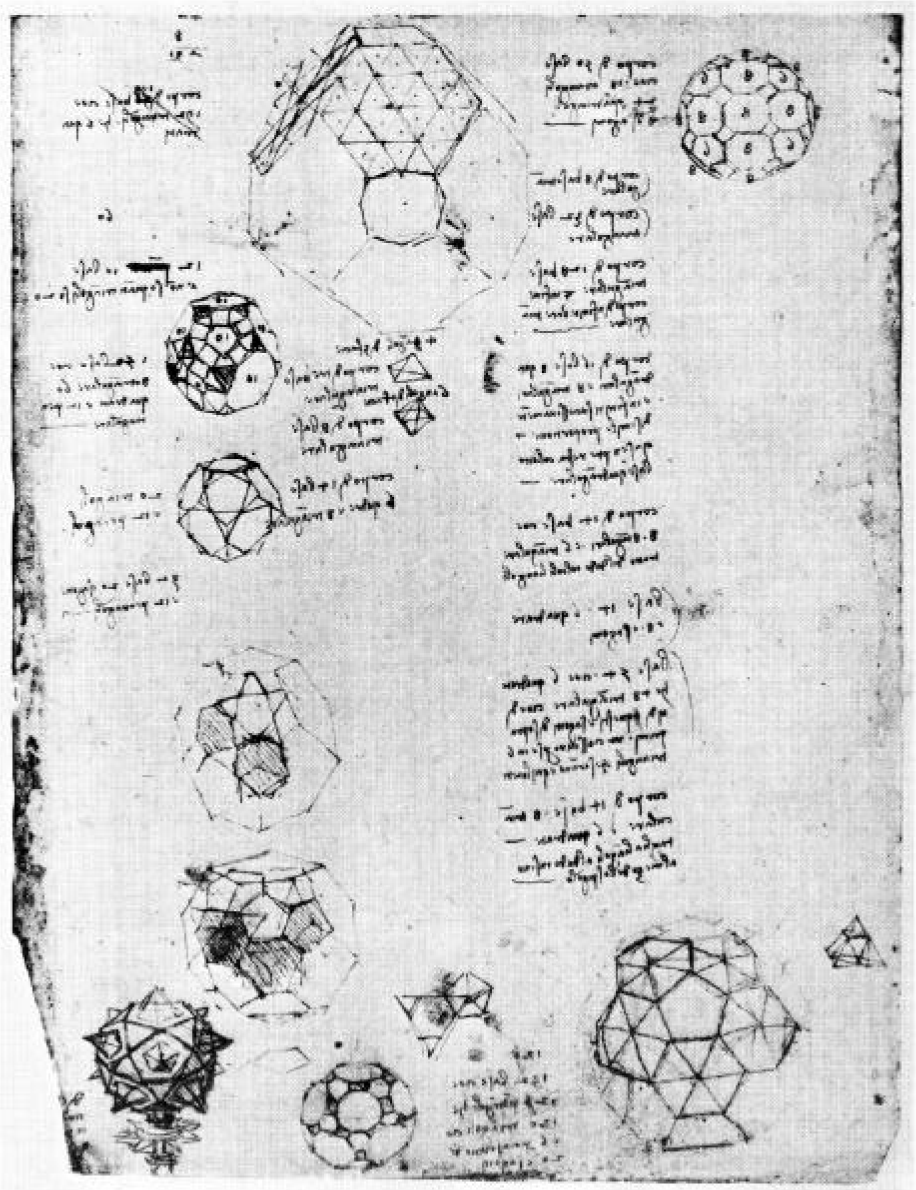
(*a*) Leonardo da Vinci, *Perspective Study of Anulus* (Milan, Biblioteca Ambrosiana, Codice atlantico, 263ʳ*a*, 706ʳ).



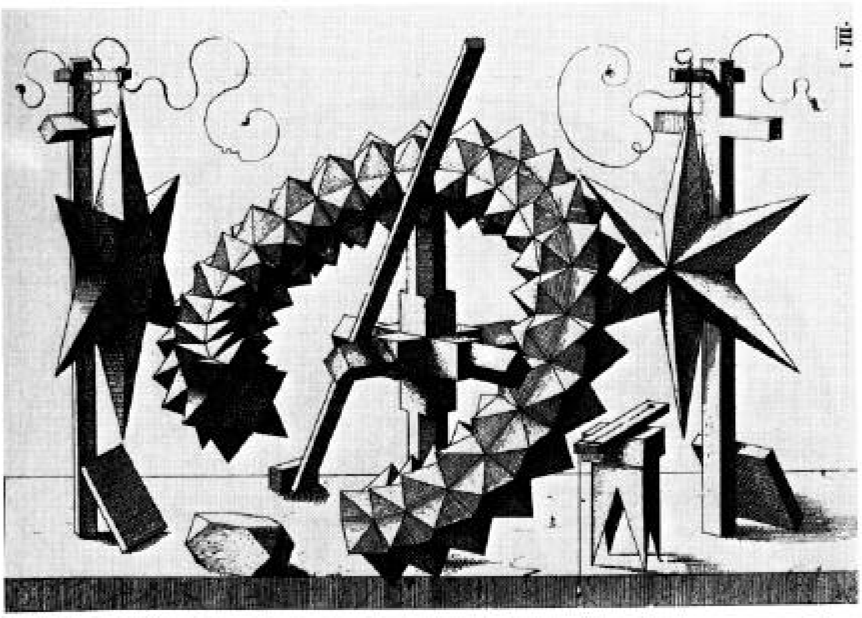
(*b*) Leonardo da Vinci, *Stellated Version of Truncated Dodecahedron*, from Luca Pacioli, *De Divina proportione* (Milan, Biblioteca Ambrosiana).

Niccolò Tartaglia, *Realm of Mathematical Sciences*, from *La Nova scientia* (Venice, 1550).

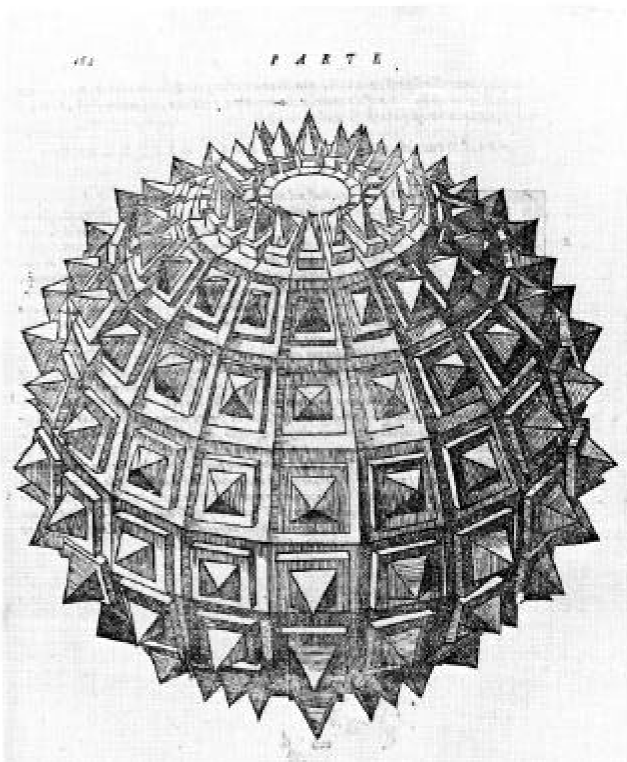
PLATE IV



Leonardo da Vinci, *Variations on Polyhedra*, including (bottom left) *a Design for a Mace*(?) (Milan, Biblioteca Ambrosiana, Codice atlantico, 272^{v}*b*, 735^{v}).

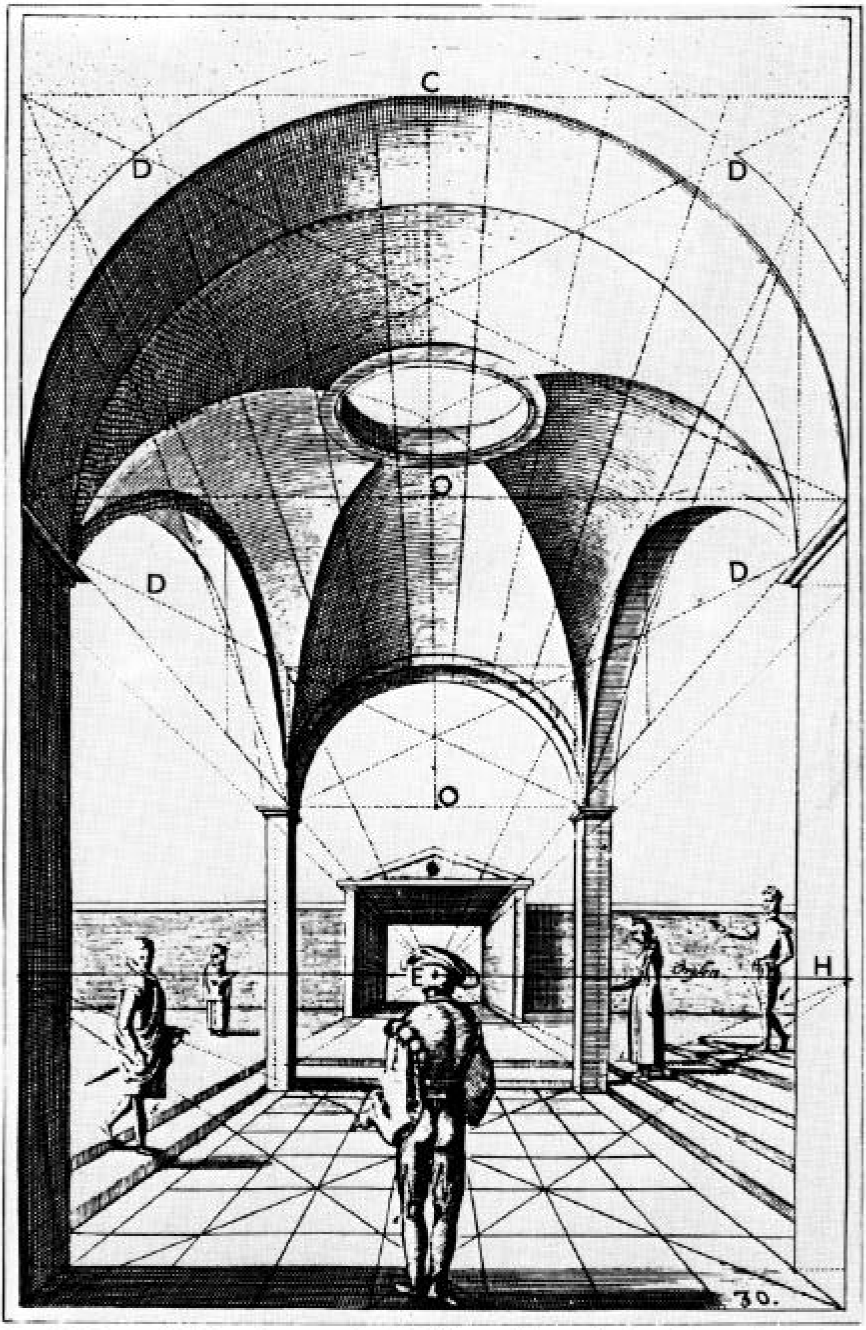


(*a*) Wenzel Jamnitzer, *Fantasia on Geometrical Bodies*, from *Perspectiva corporum regularium* (Nuremberg, 1568).

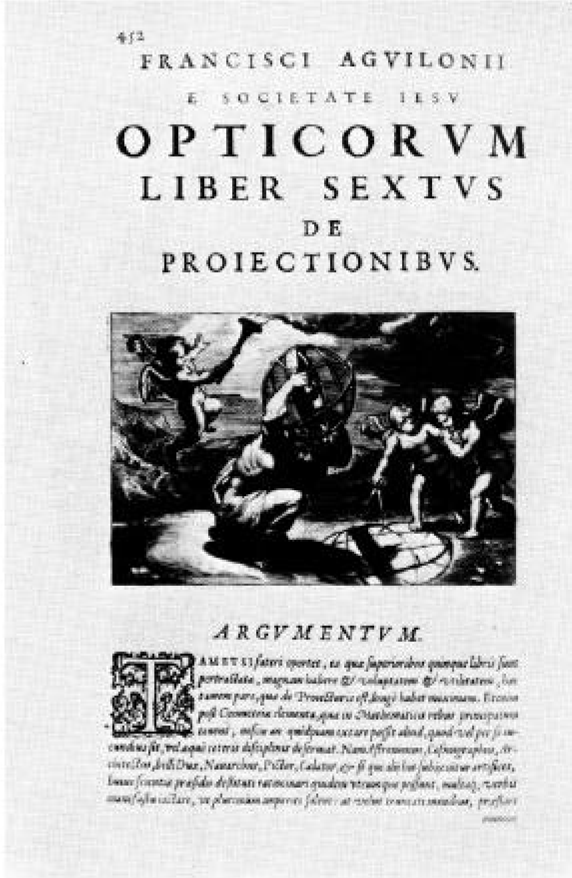


(*b*) Daniele Barbaro, *Polyhedron with Stellated Facets*, from *La pratica della perspettiva*.

PLATE VI



Jan Vredeman de Vries, *Demonstration of Architectural Perspective*, from *Perspective* (Leiden, 1604), i, pl. 28 (labelled by author to correspond to Pl. XI).


452

FRANCISCI AGVILONII

E SOCIETATE IESV

OPTICORVM

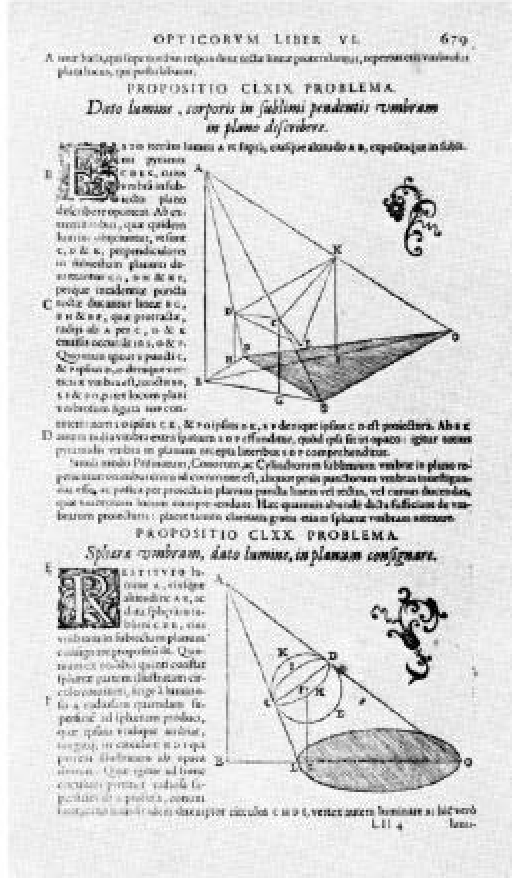
LIBER SEXTVS

DE

PROIECTIONIBVS.

ARGVMENTVM.

(a) Peter Paul Rubens, Title page of *De projectionibus*, from Franciscus Aguilonius, *Opticorum libri sex* (Antwerp, 1613).


OPTICORVM LIBER VI. 679

PROPOSITIO CLXIX. PROBLEMA.

Dato lumine, corporis in sublimi pendentis umbram in plano describere.

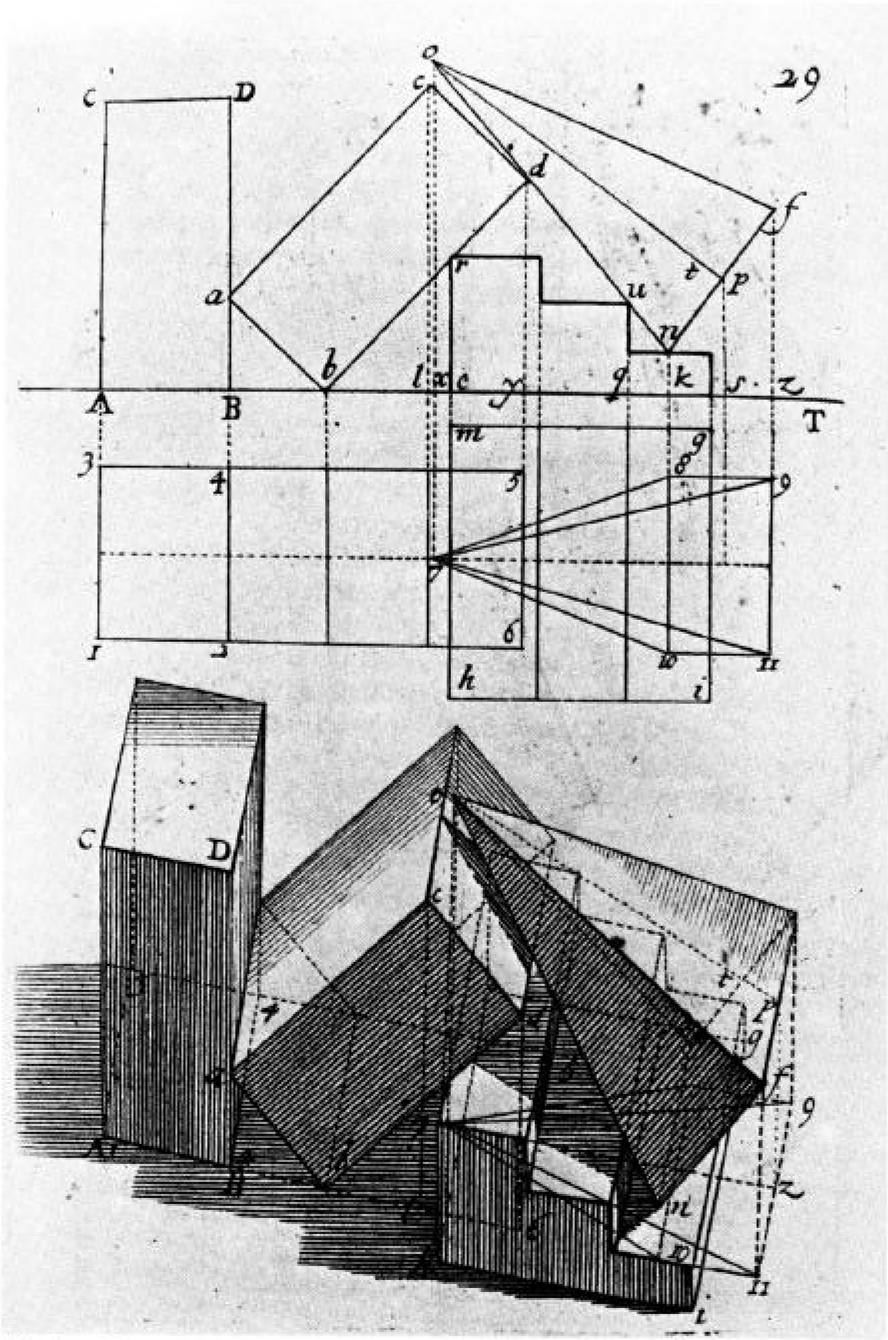
PROPOSITIO CLXX. PROBLEMA.

Sphæræ umbram, dato lumine, in planum consignare.

(b) Franciscus Aguilonius, *Shadow Projections*, from *Opticorum libri sex*.

PLATE VII

PLATE VIII



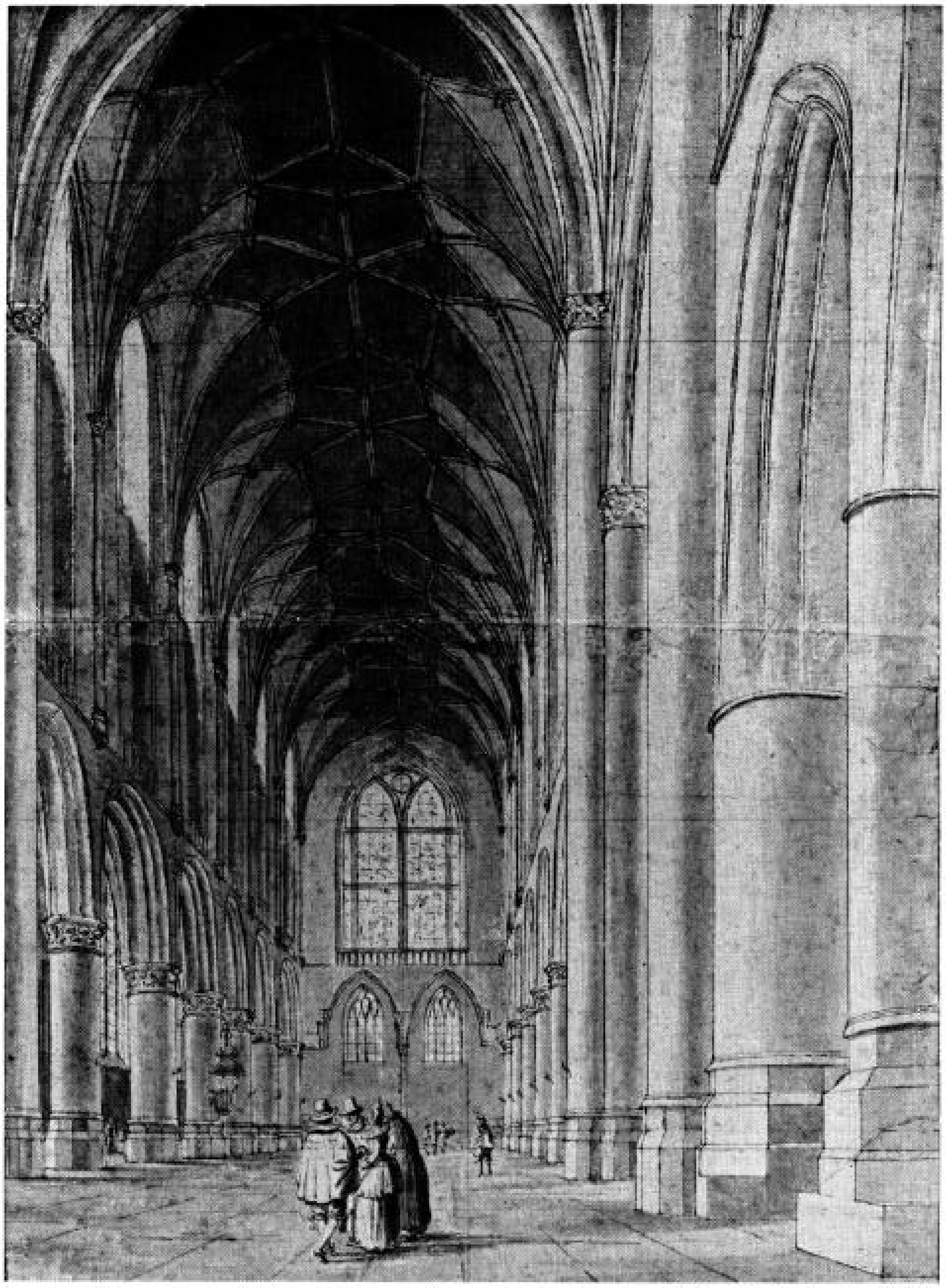
Abraham Bosse, *Perspective Study of Solid Bodies with Shadows*, from *Traite des practiques geometrales et perspectives* (Paris, 1665).

PLATE IX

The National Galleries of Scotland.

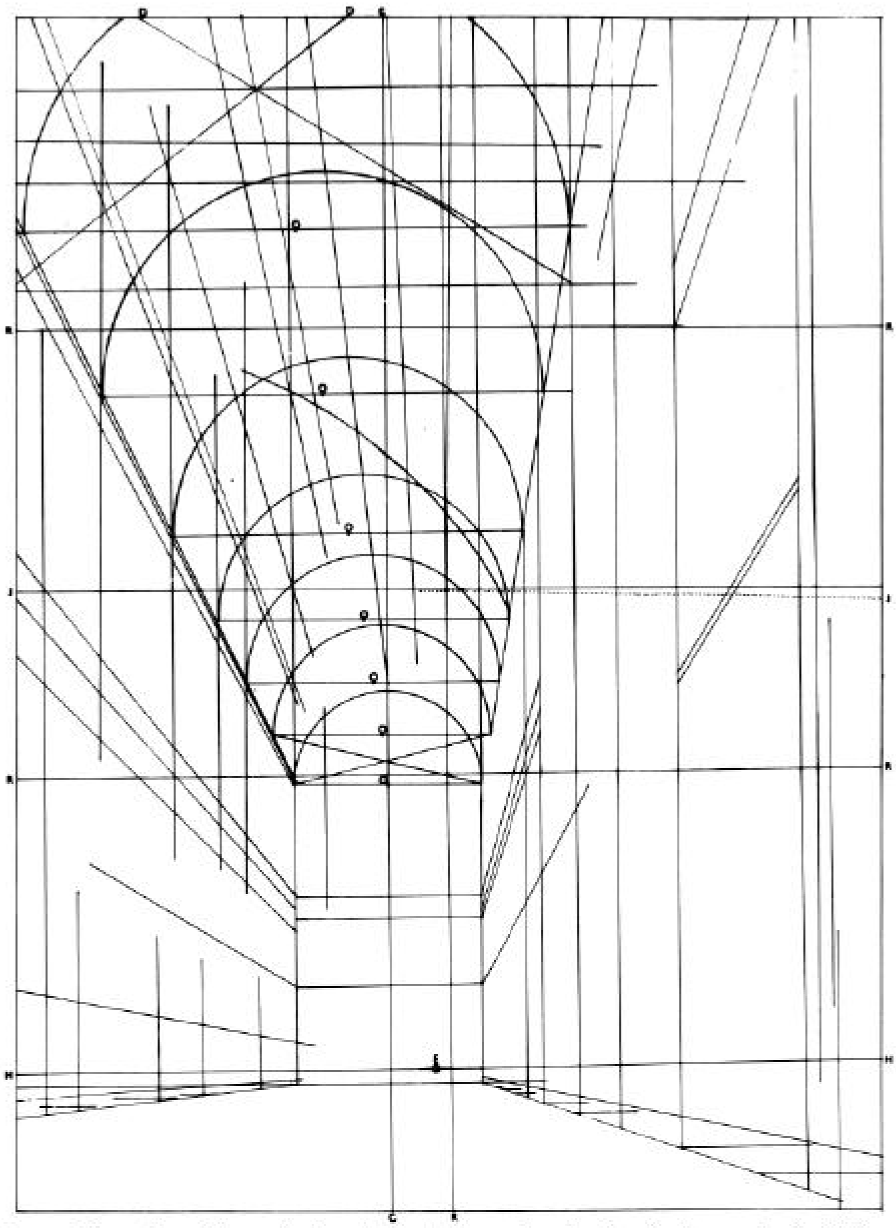
Pieter Saenredam, *Interior of St. Bravo's (known as 'Great Church') at Haarlem*, oil on oak panel, 174.8 × 143.6 cm, signed and dated 1648. Edinburgh, National Gallery of Scotland.

PLATE X



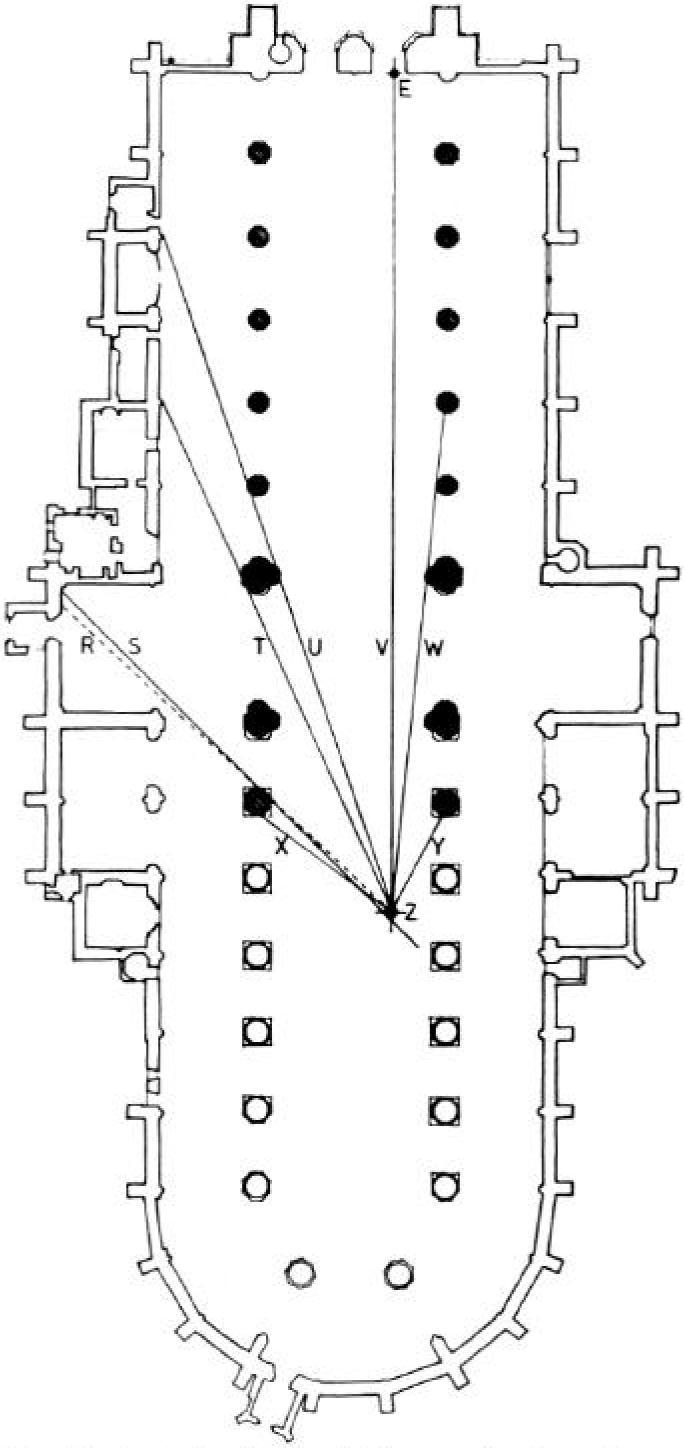
The Ian Woodner Family Collection.

Pieter Saenredam, *Right Portion of Quarter-scale Construction Drawing for Interior of St. Bravo's*, pen and ink and wash over graphite, 49.0 × 35.7 cm. New York, Woodner Family Collection.



Transcription of graphite underdrawing in construction drawing for Saenredam's *Interior of St. Bravo's* (M.K., based on full-scale colour facsimile): C, vertical line through centre of end wall; D, diagonals for construction of crossing vault; O, O, etc., compass centres for construction of vaulting arcs; E, 'eye-point'; R, squaring lines in reddish ink; J, join in sheets of paper. (Cf. Pl. VI.)

PLATE XII



Plan of St. Bravo's showing lines of sight, viewpoint: Z used for projection in Pl. XIII; R, line of sight into transept from Z; S, line of sight as in painting; V, axis of sight (corresponding to 'eye-point', E); T, U, W, typical lines of sight, indicating Saenredam's care in observing occlusions of more distant forms from the viewpoint.

Mahdad Saniee, *Perspective Projection of St. Bravo's*, based on measured plans and elevations.

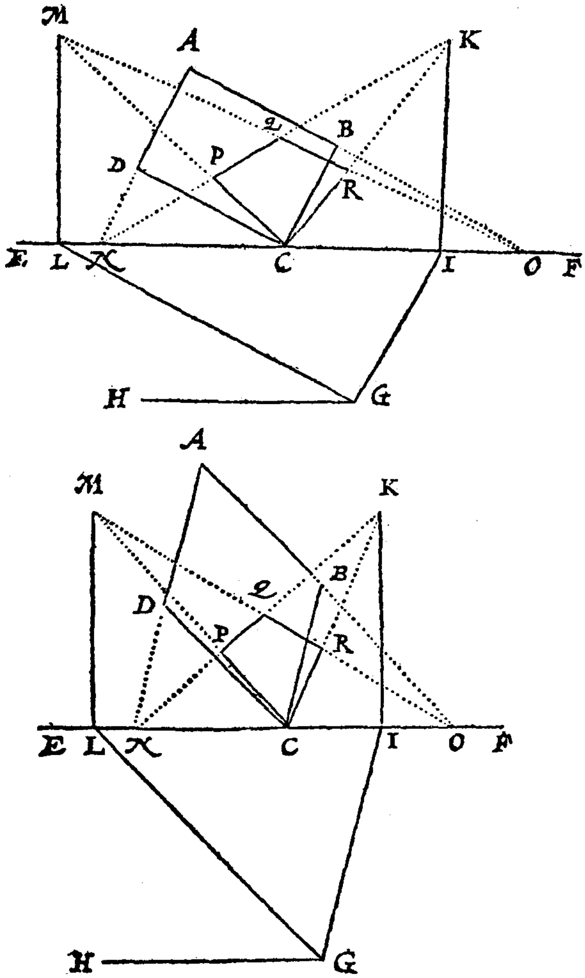


Fig. 27. Simon Stevin, *Perspective Projection of Rectangle and Parallelogram*, from *Der Deursichtighe.* Procedure as in Guidobaldo (Fig. 23, above). ABCD is projected as PQRC.

of 1614.[1] Marolois's purely mathematical demonstrations are tempered by three-dimensional diagrams of a relatively straightforward kind and detailed accounts of perspective machines. His extensive use of Hondius's engravings, including landscapes, and his illustration of Vredeman de Vries's elaborate examples of architectural perspective in action (Pl. VI) help set his mathematics in an accessible context. But, whatever his concessions in

[1] S. Marolois, *Optica sive perspectivae* in *Opera mathematica ou Œuvres Mathematiques traictons de Geometrie, Perspective, Architecture, et Fortification* (The Hague, 1614), and many subsequent editions and translations.

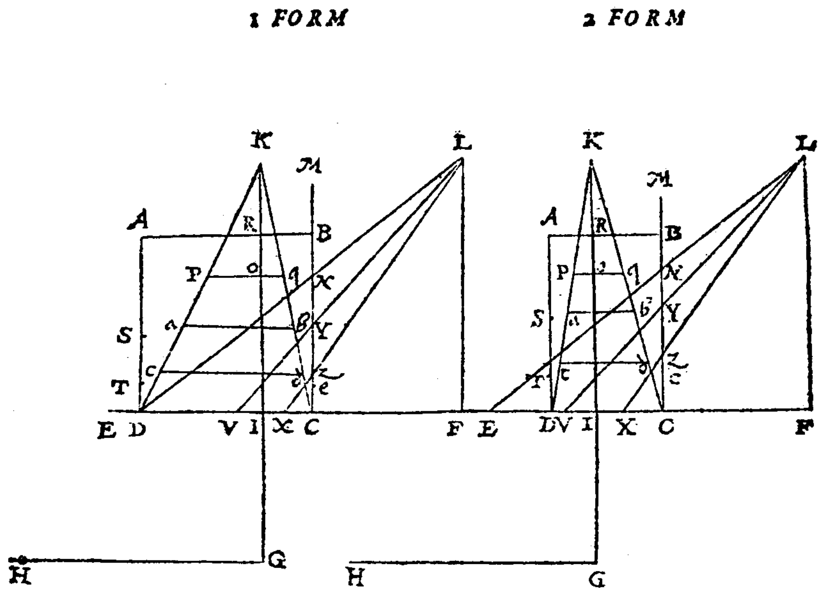


FIG. 28. Simon Stevin, *Abbreviated Perspective Constructions*, from *Der Deursichtighe*. In plan: ABCD, figure to be projected; EF, base of 'glass'; G, position of viewer; HG, height of viewer. In elevation: MC, 'glass'; F, position of viewer; LF, height of viewer. IK is drawn to equal LF and HG. D and C are joined to K. D is joined to L. Pq is drawn at level N, parallel to EF. Pqcd is the desired projection.

the direction of practice, he remained insistent that representation is a question of understanding geometrical 'rules' rather than merely recording 'what the eye receives'.[1]

I hope to suggest, at the end of this paper, that the mathematical temper of Stevin and Marolois exercised a marked influence on at least one Dutch artist of genuine importance. More immediately, however, the fortune of Guidobaldan mathematics can most readily be traced in Flanders and France. The most directly relevant episode to the story I am telling was the provision by Rubens of illustrations for François d'Aquilon's *Opticorum libri sex* of 1613.[2] There is no doubt that Rubens showed an intelligent understanding of Aguilonius's text when he designed his vignettes for the opening pages of each of the six books (Pl. VII*a*–*b*). Scholars have already pointed to clear evidence of Rubens having

[1] Marolois, op. cit., preface; A. Wheelock jnr., *Perspective, Optics, and Delft Artists around 1650* (London and New York, 1977), p. 79.

[2] F. Aguilonius, *Opticorum libri sex* (Antwerp, 1613); J. Judson and C. van de Velde, *Corpus Rubenianum Ludwig Burchard, XXI* (2 vols., Brussels, 1978), i, pp. 101–15. The Rubens illustrations seem to have been designed by 1611.

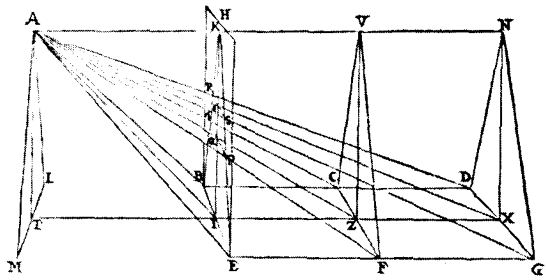


Fig. 29. Franciscus Aguilonius, *Demonstration of 'Punctum Concursus'*, from *Opticorum libri sex* (Antwerp, 1613).

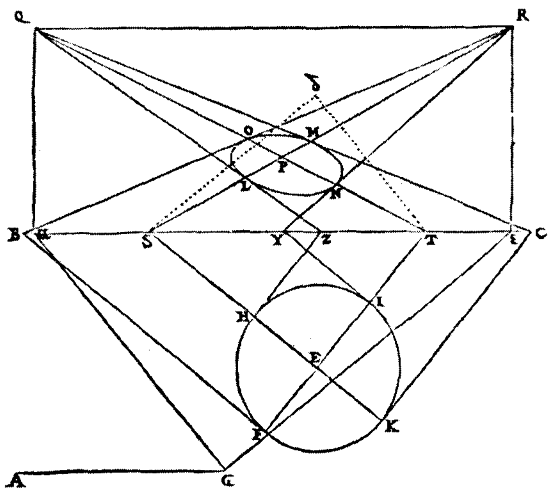


Fig. 30. Franciscus Aguilonius, *Perspective Projection of Circle*, from *Opticorum libri sex.*

taken perceptive note of the text in his paintings and drawings, and I have suggested elsewhere that Aguilonius's colour theory may in its turn have responded to Rubens's experience as an artist.[1] Aguilonius's purely geometrical diagrams, which often make close reference to Guidobaldo's treatise (Figs. 29–30), are most unlikely to have been of directly practical use to Rubens, but would have appealed to his undoubtedly strong sense that art was ultimately related to a set of underlying principles.

[1] C. Parkhurst, 'Aguilonius' Optics and Rubens' Colour', *Neederlands Kunsthistorisch Jaarboek*, xii (1961), 34–50; M. Jaffé, 'Rubens and Optics', *Journal Warburg and Courtauld Institutes*, xxxiv (1971), 362–5; J. Held, 'Rubens and Aguilonius; New Points of Contact', *Art Bulletin*, lxi (1979), 257–64; M. Kemp, 'Yellow, Red and Blue: the Limits of Colour Science in Painting' in A. Ellenius (ed.), *The Natural Sciences and the Arts* (Acta Universitatis Upsaliensis, xxii, Uppsala, 1985), pp. 98–105.

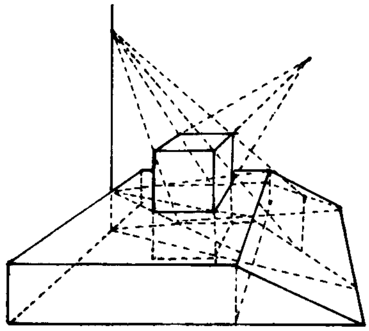


FIG. 31. Matteo Zaccolini, *Geometrical Shadow Projection* (M.K., based on *Della Descrittione delle ombre* (Florence, Laurentian Library, MS Ash 1212[1]).

In his actual paintings these principles play a genuinely 'underlying' rather than an overtly obvious role, and it is rather to his great contemporary, Nicolas Poussin, that we look for the more direct translation of visual geometry on to canvas. His Berlin *Self Portrait*, in which he prominently displays an optical treatise entitled *De lumine et colore*, is an open declaration of his aspirations. The implication is that Poussin himself was to compse a treatise on the subject, though there is no evidence that he ever did so. The title recalls those of the four manuscripts on light and colour by Matteo Zaccolini which Cassiano del Pozzo owned in copies commissioned for himself, and we know that one of them was specifically recopied for Poussin before 1640. That particular manuscript deals very ingeniously with the science of geometrical shadow projection (Fig. 31).[1] Such geometrical concerns may well have been later reinforced in Poussin's Roman circle by the work of Athanasius Kircher, particularly his *Ars magna lucis et umbrae.*[2] Kircher, in a book laced with neo-Platonic mysticism, provided a wide compendium of visual delights, from magic lanterns to geometrical diagrams in the Guidobaldan-Aguilonian manner. His analysis of planes in a solid figure (Fig. 32) makes a particularly good comparison with planar structures in Poussin's paintings. If

[1] E. Cropper, 'Poussin and Leonardo: Evidence from the Zaccolini Manuscripts', *Art Bulletin*, lxii (1980), 570–82; A. Blunt, *The Paintings of Nicolas Poussin, A Critical Catalogue* (London, 1966), no. 1, p. 7; J. Bell, 'Colour and Theory in Seicento Art: Zaccolini's *Prospettiva del colore* and the Heritage of Leonardo' (Ph.D. thesis, Brown University, 1983, University Microfilms 8325951).

[2] A. Kircher, *Ars magna lucis et umbrae* (Rome, 1646). For suggestions regarding Kircher's Aguilonian colour theory and Poussin's late paintings see Kemp, 'Yellow, Red and Blue . . .'.

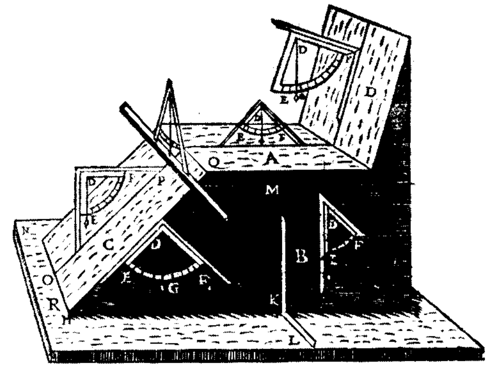


FIG. 32. Athanasius Kircher, *Analysis of Planes*, from *Ars magna lucis et umbrae* (Rome, 1646).

we add that Pietro Testa, a painter and engraver friendly with Poussin, grappled with the *Analemma* of Ptolemy as described in Daniele Barbaro's *Vitruvius*, we can come close to confirming that Poussin was fully aware of the intellectual nexus within which *perspectiva* was situated.[1] For good measure, there is evidence that Poussin himself studied Alhazen, and probably also Witelo, presumably in Risner's edition.[2]

However, just as it seems that Guidobaldan mathematics might be brought into some degree of contact with the practice of art, the science of three-dimensional geometry was simultaneously being extended further in a technically mathematical direction by Girard Desargues of Lyons. A genuinely original if eccentric mathematician, he was well respected by both Pascal and Descartes, the latter of whom visited him while he was working on fortifications at the siege of La Rochelle in 1628. His work on conics, the picturesquely entitled 'Rough Draft of an Attempt to Deal with the Outcome of a Meeting of a Cone with a Plane', helped sow the seeds of non-Euclidian geometry, but was only to be fully taken up by Poncelet in the early nineteenth century.[3] Vital steps in the development of new postulates appear to have been taken independently by Kepler and Desargues. The new geometry challenged central assumptions of Euclidian theory. Straight lines came to be interpreted as equivalent to circles which

[1] E. Cropper, 'Virtue's Wintry Reward: Pietro Testa's Etchings of the Seasons', *Journal Warburg and Courtauld Institutes*, xxxvii (1974), 257, note 35.

[2] Blunt, *Nicolas Poussin*, pp. 224ff. and 372, note 3.

[3] G. Desargues, *Brouillon project d'une atteinte aux événens des rencontres d'une cone avec un plan* (Paris, 1639) in *Œuvres de Desargues* (ed. M. Poudra, 2 vols., Paris, 1864), i, pp. 103–230. See J. V. Field and J. J. Gray, *The Geometrical Work of Girard Desargues* (forthcoming).

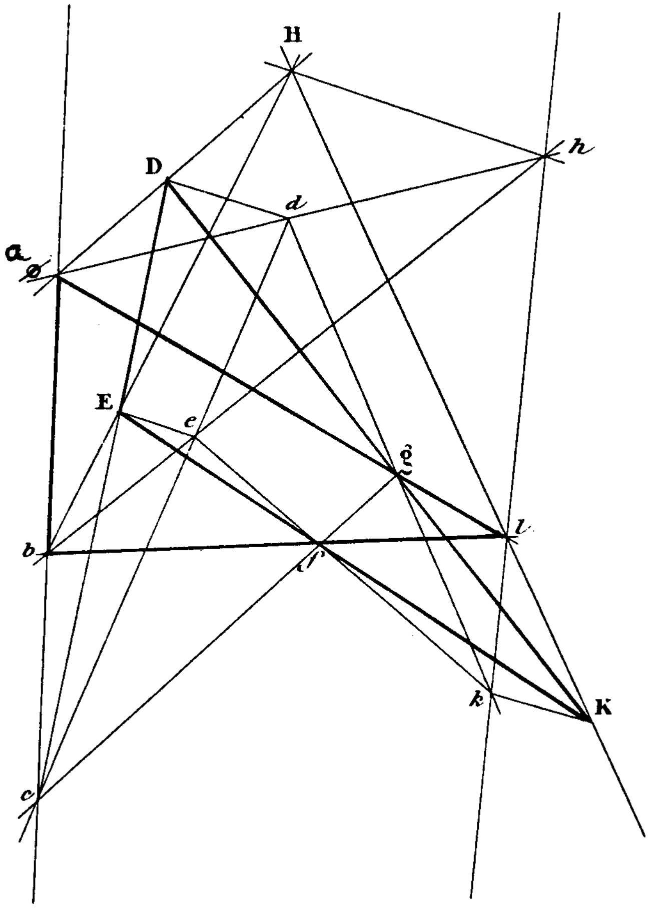


FIG. 33. Girard Desargues, '*First Geometrical Proposition*', from *Méthode universelle* (1636), in Abraham Bosse, *Manière universelle de Mr. Desargues* . . . (Paris, 1648). Exploring the relationship between the triangles DEK and abl. (Note: in the printed diagram a is wrongly labelled o.)

possess radiuses of infinite length, and parallel lines regarded as meeting at infinity. Although I do not believe that painters' perspective in itself was more likely to engender an infinite view of space than a finite view, it is not hard to see how Desargues could regard the *punctum concursus* of Guidobaldo as helping to engender a geometry embodying alternative principles.

Desargues's own writings on perspective, beginning with a short tract published in 1636, were cast in the guise of his 'Manière (or 'Méthode') universelle'.[1] During the course of his search for

[1] Poudra, op. cit., i, pp. 53–4. Desargues's pamphlet of 1636 is best known through A. Bosse, *Manière universelle de Mr. Desargues pour pratiquer la perspective*

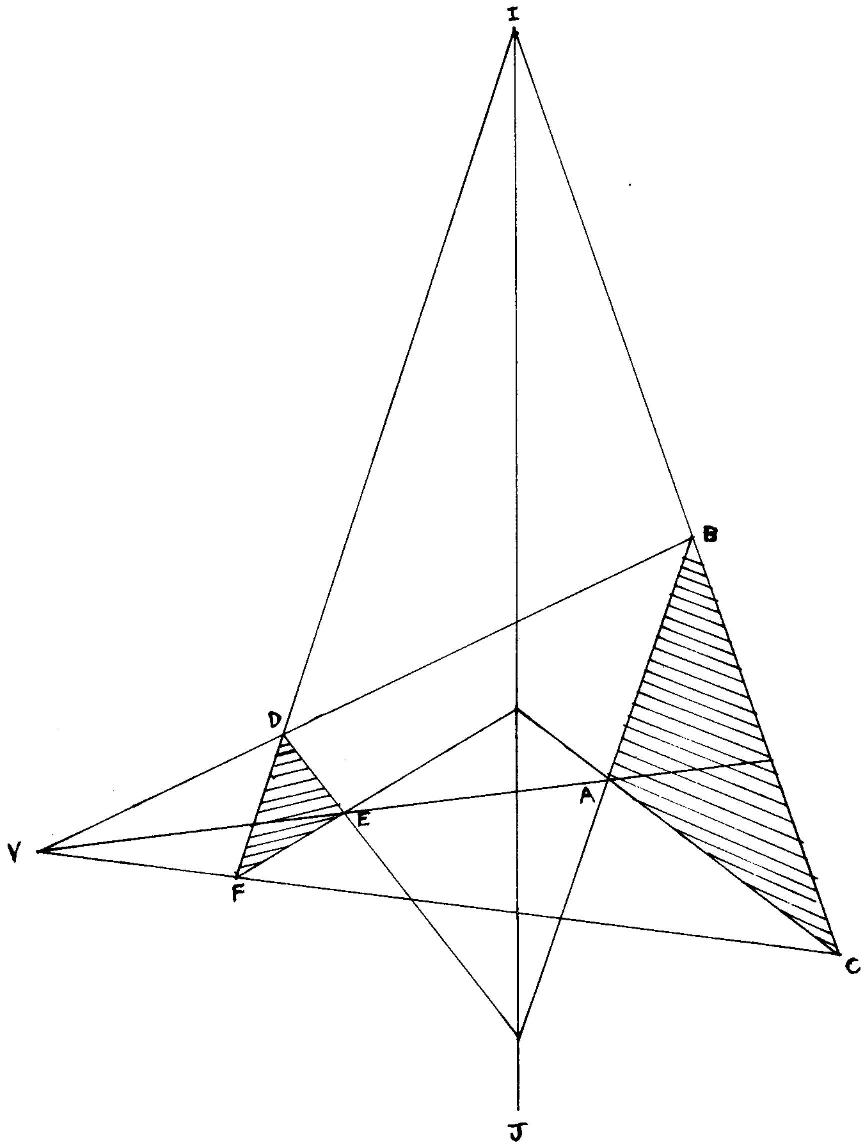


FIG. 34. Desargues's Theorem (M.K.). When the lines joining the vertices B and D, A and E, C and F converge to a single point (V), the extensions of opposite sides of the triangles meet on a common line (IJ).

(Paris, 1648). Judith Field has kindly drawn my attention to the fact that a copy of the pamphlet survives in the Bibliothèque Nationale, Paris. See W. Ivins jnr., 'A Note of Gerard Desargues', *Scripta Mathematica*, ix (1943), 33–48, and xiii (1947), 203–10, and *Art and Geometry* (Cambridge, Mass., 1966); N. A. Court, 'Desargues and His Strange Theorem', *Scripta Mathematica*, xx (1954), 5–13 and 155–64.

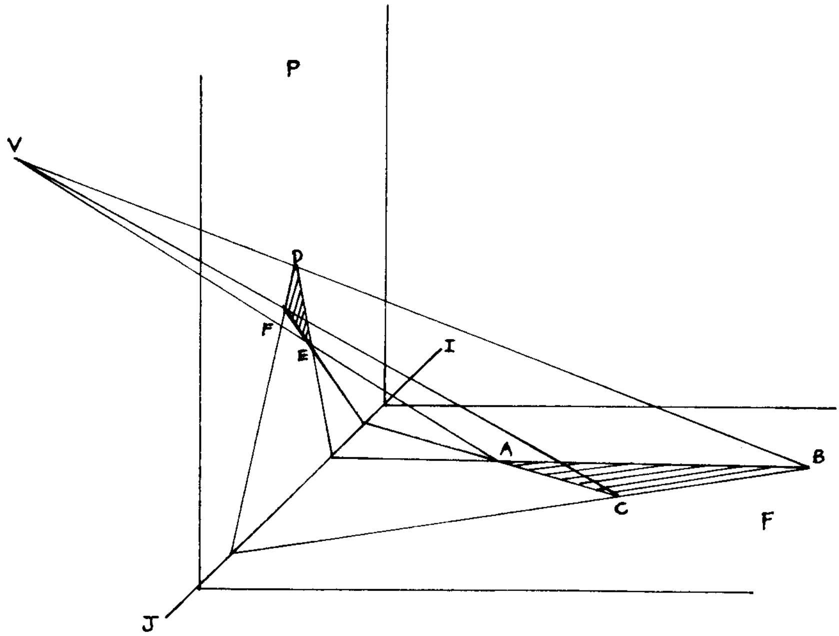


FIG. 35. Perspective Projection of Triangle (M.K.), to correspond to Fig. 34 above: V, eye; IJ, junction of picture plane (P) and ground plane (F). ABC is seen as DEF.

all-embracing rules of projection, he formulated the fundamental theorem which bears his name. This states that if two triangles (Figs. 33–4) are arranged in such a manner that lines connecting pairs of corresponding vertices converge to a point, then the points of intersection of pairs of corresponding sides lie on a straight line—and conversely. Desargues's own geometrical demonstration demands a persistence and geometrical aptitude such as to discourage most artists from working out the connection of his theorem with the foreshortening of a given triangle in a standard perspective analysis (Fig. 35).

Desargues, as an architect and engineer, was in no doubt that his mathematics intersected with practical matters: 'I freely avow that I never had a taste for study or research, in either physics or geometry, except in so far as they could serve the fundamental purpose of arriving at a form of knowledge of proximate causes, . . . having observed that a large part of the practice of the arts is founded on geometry as a basis of certainty, amongst others the art of cutting of stone in architecture, . . . that of sundials, . . . that of perspective . . .'[1] His own treatise on the art of stone-cutting

[1] Poudra, op. cit., i, pp. 24–5.

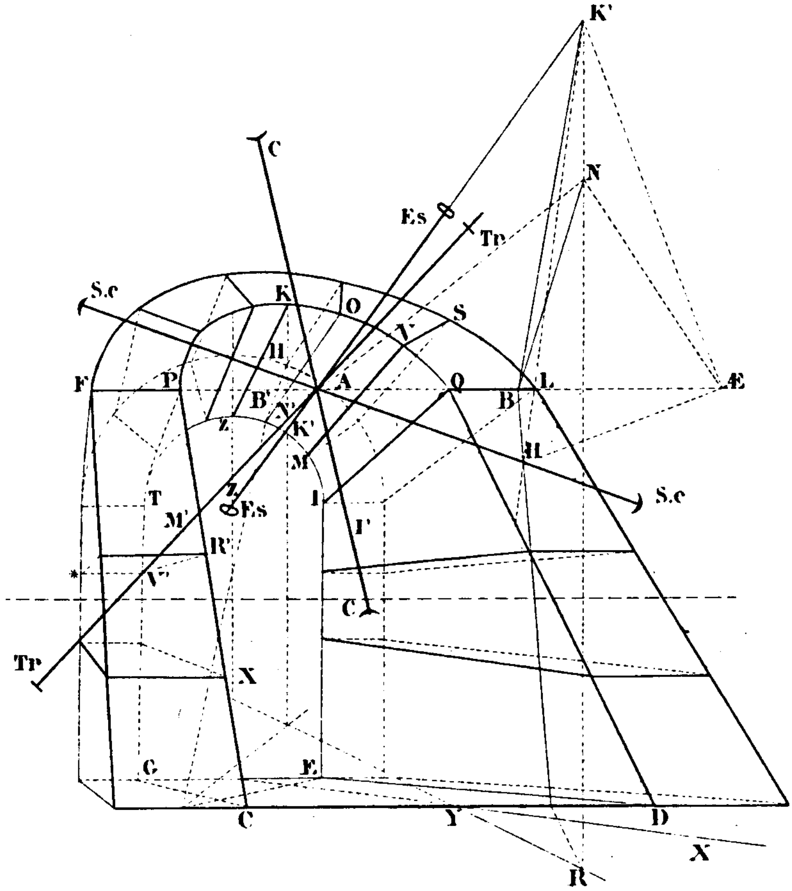


FIG. 36. Girard Desargues, *Perspectival Demonstration of Blocks to Construct Canted Arch*, from *La Coupe des pierres* . . . (Paris, 1640).

(Fig. 36) illustrated the relevance of three-dimensional geometry to the construction of complex forms in actual architecture. His own highly individual and literally picturesque vocabulary of mathematical terms, sprinkled with botanical terms such as 'trunk', 'tree', 'branch', and 'palm', underlines his sense of contact between abstract geometry and physical reality.[1]

However, although a small band of devotees including Laurent de la Hyre and Eustache le Sueur were prepared to follow or at least acknowledge Desargues's lead, most practitioners in the various fields touched by his science seem to have reacted adversely, if at all. His ideas were actively ridiculed in the

[1] Ibid., 99–102 for a glossary of Desargues's terms.

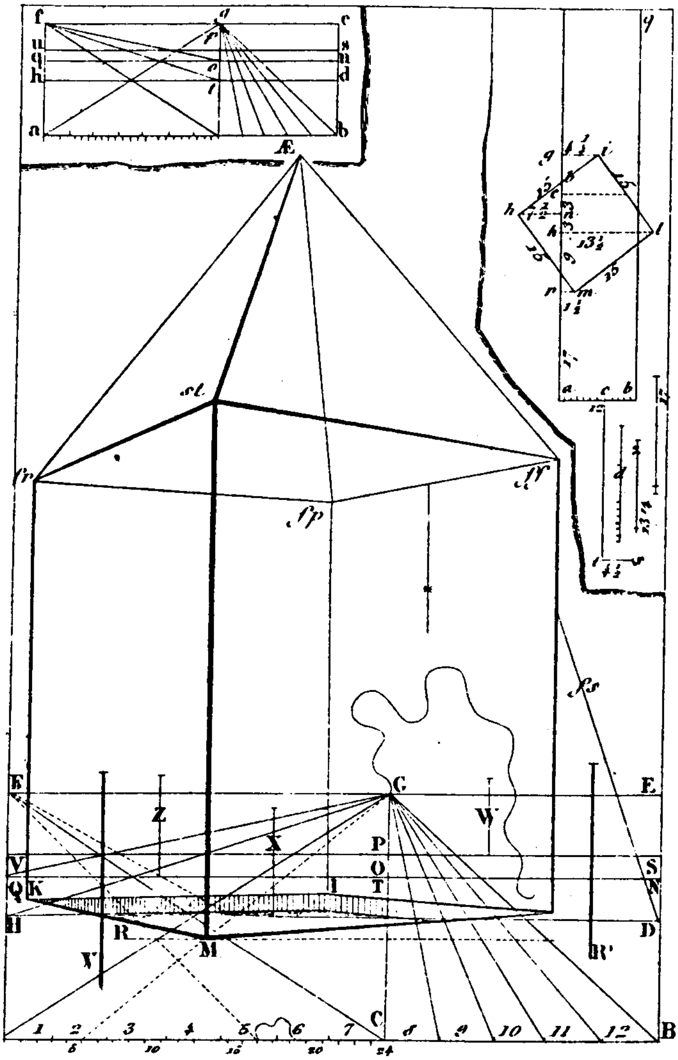


FIG. 37. Girard Desargues, *Metrical Perspective Construction for Painters*, from Abraham Bosse, *Manière universelle* . . . Inset drawing upper right: scale plan of object, picture plane, and viewpoint. Main drawing—1. Base line AB is divided into 12 feet corresponding to ab in inset drawing upper right. 2. FE = horizon, on which G marks the axis of sight. 3. Diagonals AG and FC give horizontal HD. Inset drawing upper left—4. Draw ft, giving horizontal qn where ft intersects ag. 5. Draw fo, giving horizontal us, and so on . . . Main drawing—6. AC is divided into 24 feet corresponding to tc (viewing distance) in inset drawing upper right. 7. HD (see 3 above) is 24 feet behind picture plane. 8. QN is 48 feet behind picture plane, and so on . . . 9. BG and orthogonals to right of GC are drawn. 10. Each intersection of orthogonals on HD, QN, etc. = scale of 1 foot. Thus HD contains 24 scale feet, QN 36 scale feet, etc. To find the position of m—11. A

influential circle of Jacques Dubreuil, whose own treatise shared Marolois's concern to give practical guidance to practitioners rather than blinding them with science.[1] Desargues's ideas were championed in print and in teaching at the Academy by Abraham Bosse, who amplified the highly condensed demonstration of pictorial perspective given by Desargues himself (Fig. 37) with step-by-step explanations and accompanying diagrams of perspective in action (Pl. VIII).[2] However, Bosse's position in the Academy became increasingly strained, and a final breach occurred in 1661. It is a measure of the Academicians' distrust of Desargues's advanced geometry, that le Brun favoured the technique of le Bicheur (Fig. 38), which is little more than the skeleton of the old distance-point method in a Desargues-like uniform.[3]

Looking at the work of Guidobaldo, Stevin, and Desargues in the period from 1600, we can see the extent to which the geometrical core of their work had come to lie outside the competence and interests of artists in general. We would not expect painters to resort to Guidobaldo's geometrical proofs,

[1] (J. Dubreiul) *La Perspective pratique necessaire à tous peintres par un Parisien religieux de la Compagnie de Jesus* (Paris, 1642), trs. E. Chambers, *The Practice of Perspective: or an easy Method of Representing Natural Objects According to the Rules of Art . . . by a Jesuit of Paris* (London, 1726).

[2] Bosse, op. cit.; and *Moyen universal de pratiquer la perspective sur les tableaux ou surface irrégulières* (Paris, 1653); and *Traité des pratiques géométrales et perspectives* (Paris, 1665); and *Le Peintre couverty aux précises et universelles regles de son art* (Paris, 1666); and *Traité de leçons* (Paris, 1665).

[3] J. le Bicheur, *Traicté de perspective faict par un peintre de l'academie royale. Dedié à Monsieur le Brun, première peintre du roy* (Paris, 1657). C. Goldstein, 'Studies in Seventeenth-Century French Art Theory and Ceiling Painting', *Art Bulletin*, xlvii (1965), 231–56.

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distance equivalent to ra (upper right inset) is measured on the scale AC, i.e. at point 17. 12. Point 17 is joined to E, and the point at which it intersects AG provides the horizontal R on which M is to be located. 13. A distance equivalent to rm (upper right inset) is measured on the horizontal R using the scale of orthogonals to right of GC. 14. The scaled distance is measured from the intersection of AG and R to give the location of M. 15. All other points on the plan can be produced by procedures equivalent to 11–14 above. The verticals, V, Z, W, R^1, represent the height of standing figures. The height of the verticals of the house are calculated according to the corresponding horizontal scale.

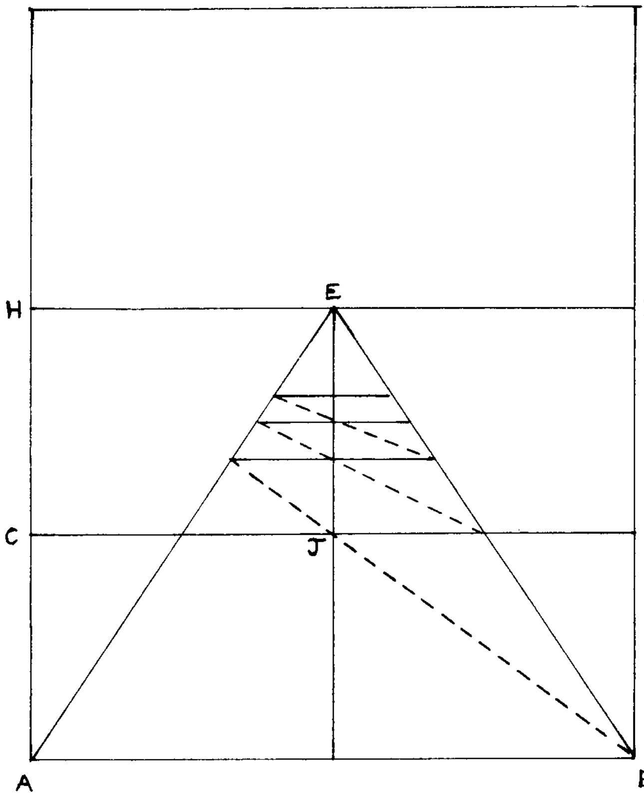

FIG. 38. Jacques le Bicheur, *Perspective Construction* (M.K., based on *Traicté de perspective* . . ., Paris, 1657): HI, horizon; E, 'vanishing point'; CD, first horizontal division, such that ID = DB. The extension of the diagonal BJ given the second horizontal, and so on.

Stevin's principles of homology or Desargues's theorem to construct their illusions. The best for which we can hope from a geometrically minded artist in the seventeenth century is an awareness of how the science of artificial perspective in picture-making was a practical, limited manifestation of abstract theorems in projective geometry.

Did such artists exist, or did the growing gap between the needs of painters and the deductions of mathematicians effectively debar the kind of balance between mathematics and natural observation which had existed in fifteenth-century art? Could there, in other words, be a Baroque Piero della Francesca? I have already suggested that Poussin may in certain respects fulfil such a role, but I would not wish to push this claim too far, given our lack of sustained, first-hand evidence of his views on such technical matters. Rather, I should like in conclusion to propose that Pieter

Saenredam is the painter to whom we may most immediately look for a knowing expression of the new mathematics in art.

We have recently learnt at least some of the contents of Saenredam's library.[1] In addition to editions of Vitruvius, Dürer, Serlio, and Scamozzi, all of which treat in whole or in part the geometry of visual beauty, he possessed a series of important texts on mathematics, including three Euclids. He owned a range of Stevin's writings in pure and applied mathematics, one volume of which was annotated by Pieter Wils, a mathematician, astronomer, surveyor and military engineer, with whom Saenredam had been associated since about 1628. Stevin's *De Sciagraphia* is not on the surviving list, but this makes little difference; the flavour of his interests is clear.

Work on which Hugh Macandrew and I have recently been engaged at the National Gallery of Scotland—in collaboration with an architect, Mahdad Saniee, and the Keeper of Conservation, John Dick—has revealed the astonishing balance of geometrical precision, artful manipulation and intuitive observation which lies behind Saenredam's large-scale painting of the *Interior of St. Bavo's, Haarlem* (Pl. IX).[2] Analysis of the graphite underdrawing of one of the two surviving portions of the quarter-scale *modello* (Pls. X–XI) has revealed a complex network of construction lines not dissimilar to those used by Vredeman de Vries (Fig. 36), but with a far greater awareness and exploitation of a particular viewpoint. On the other portion of the *modello* the distance point is clearly marked on the pier at the extreme left, and its distance from the central axis—30 Dutch feet—is duly recorded.

In this attention to the distance point we may see the direct benefit of Saenredam's mathematical understanding of perspective. The geometrical perspectivists such as Stevin *began* their constructions with the definition of the relationships between the object, plane, and viewing position, whereas the artist's handbooks concentrated upon a practical means of establishing a spatial pattern in which the viewing distance was often more implicit than explicit. Saenredam's awareness of viewpoint was an essential step in his developing a heightened sense of the transformation

[1] R. Ruurs, 'Pieter Saenredam: zijn boekenbezit en zijn relatie met der landmeter Pieter Wils', *Oud Holland*, xcvii (1983), 59–68.

[2] For the detailed evidence see H. Macandrew (ed.), *Dutch Church Painters. Saenredam's 'Great Church at Haarlem' in Context* (Exhibition catalogue, Edinburgh, 1982). See also M. Kemp, 'Simon Stevin and Pieter Saenredam: a Study of Mathematics and Vision in Dutch Science and Art', *Art Bulletin*, forthcoming.

of three-dimensional forms into *flat* shapes on the picture plane. The result is that the forms both describe space and articulate the surface of his design.

I should stress, however, that his masterpiece is not a pure exercise in projective geometry. A careful perspective projection (Pl. XII) from an appropriate viewpoint (Pl. XIII), made by Mahdad Saniee using accurate plans and elevations of the church, displays so many significant conjunctions with the painting as to indicate that the painter used the survey measurements which we know to have been available to him.[1] No less revealing, however, are his departures from absolute accuracy, above all in his manipulation of the vertical proportions in certain places to accentuate the sense of soaring height. His significant raising of the apexes of the crossing arches—extending their height above the capitals of the crossing by up to one half—provides the subjective effect required and contributes a major component in the compositional sense of picture surface. At the same time, crystalline light glances with sensuous subtlety from convex, concave, and planar surfaces, without detracting from the interplay of spatial and surface geometry.

This painting alone is enough to stress that, whatever the changing relationship between the geometrical theory and painter's practice in different periods, it remained feasible if increasingly difficult for a great artist to effect a miraculous reconciliation between the abstract technicalities of mathematics and the sensuous perception of nature. After 1600 this reconciliation was necessarily different from that in the Renaissance. In the exceptional hands of a Saenredam—and I suspect we could include Vermeer—geometrical perspective was neither an end in itself, nor simply a means to an end, but could function as an integral component in the unfailingly complex dialogue between the mind and reality, as expressed in a particular manner in the visual arts.

[1] R. Ruurs, in his review of the Edinburgh exhibition in *Oud Holland* (1985), pp. 161–4, shows that Saenredam used the measurements by Pieter Wils in S. Ampzing, *Beschrijvinge ende lof der Stad Haerlem in Holland* (Haarlem, 1628), appendix.