Everybody Counts but Not Everybody Understands Numbers

Professor Brian Butterworth FBA *delivered the annual Joint British Academy/British Psychological Society lecture on 16 November 2004. In the edited extract below, he describes the unrecognised handicap of dyscalculia.*

DYSCALCULIA is a congenital condition that prevents people learning arithmetic. It is a serious handicap – at least as serious as dyslexia. It affects, according to our current best prevalence estimates, more than 5% of the population (that is, more than 3 million citizens of the UK alone).

The most usual lay and educational explanation of why children have severe difficulty in learning numerical skills is stupidity, or in a more sophisticated variant, poor reasoning ability. Conversely, there are many examples of people with generally very low cognitive abilities who are very skilled at calculation – those with 'savant skills'. These findings suggest that good reasoning is neither necessary nor sufficient for high levels of mathematical skills.

In general the career consequences of low maths ability are considerable. According to a study for the Basic Skills Agency, poor numeracy is more of a handicap than poor literacy in getting a job, keeping a job and getting a promotion. In a study described in more detail below, one nine-year old with very low arithmetical skills was clearly aware of this. He told us, 'If you don't learn, yeah, you won't have a good job and you'll be a dustbin man.'

Of course, it is possible to reach high office without good numerical skills. The former Chief Inspector of Schools, Chris Woodhead, in a radio discussion of maths teaching was asked to calculate half of three-quarters. He refused to answer. A spokesman later said, 'Chris did know, but didn't want to answer. Anyway, his speciality was English not mathematics.' Perhaps he was recalling what happened to Stephen Byers. As an education minister, he was responsible for introducing the new National Numeracy Strategy, which entailed memorisation of multiplication tables. In a radio interview, he was asked, 'Mr Byers, what is 8 x 7?' He answered '54'. He didn't last much longer at Education. He was transferred to the Treasury.

Poor arithmetical skills also have emotional consequences. Anna Bevan and I carried out a study with eight- to nine-year old children for the DfES. Our original purpose was to assess children's attitudes to the new National Numeracy Strategy, to maths lessons, and more particularly why some children showed 'maths anxiety'. There is a Maths Anxiety Rating Scale, but this is just a measurement tool, not an explanation. We thought the best approach would be to ask the children. However, a one-on-one interview between a Large Teacher-like Person (LTLP) and a small pupil-like person seemed to us likely to elicit the kind of answers the child has learned the LTLP wants and expects. So instead we organised the students into five child focus groups with a moderator, Anna, who would lead the discussion. We also divided the children according to their maths ability, so those with low ability would be able to share their problems, and those with average and good abilities could discuss theirs. We were surprised, and shocked, by what the children said. The following verbatim transcripts from two groups illustrate the common experience of dyscalculic children: falling behind the rest of the class and feeling isolated, stupid and distressed, as a consequence:

Moderator: How does it make people feel in a maths lesson when they lose track?

Child 1: Horrible

Moderator: Horrible? Why's that?

Child 1: I don't know

Child 3: (whispers) He does know

Moderator: Just a guess

Child 1: You feel stupid

(School 5)

Child 5: It makes me feel left out, sometimes.

Child 2: Yeah.

Child 5: When I like – when I don't know something, I wish that I was like a clever person and I blame it on myself –

Child 4: I would cry and I wish I was at home with my mum and it would be – I won't have to do any maths –

(School 3)

And these children, we learned, are teased and stigmatised by their peers. Since the National Numeracy Strategy requires a daily maths lesson with whole class teaching, there can be daily humiliation or distress.

Now, mathematics, even in the first grades of schooling, is made up of a wide variety of skills. These include being able to read and write numbers, understanding number words, counting, estimating, retrieving arithmetical facts (number bonds, multiplication tables), understanding arithmetical laws such as commutativity of addition and multiplication (but not subtraction and division), knowing the procedures for carrying and borrowing in multi-digit tasks, being able to solve novel word problems, and so on. To solve many arithmetical problems may require recruiting several of these skills, each of which may go wrong in several ways. On tests of arithmetic, any error will lead simply to nul points on that question. Therefore very different cognitive problems can lead to similar outcomes on standardized tests of arithmetical ability. And of course, there are many other reasons, besides cognitive deficits, for performing poorly on tests of arithmetic, among them behavioural problems, anxiety, missing lessons, and inappropriate teaching.

Given the complexity of even early school arithmetic, and the range of symptoms, it is not surprising that investigators have come up with a variety of theories as to why children have difficulty learning arithmetic. These try to explain the problem in terms of some other, more general or more 'basic', cognitive capacity, such as memory, spatial ability, language, or reasoning ability. All of these theories are highly plausible. But there is another.

Numerosity – the core number concept

Piaget, following the philosophers of mathematics with whom he worked, distinguishes between cardinal numbers and ordinal numbers. Cardinals denote the number of objects in a set. Thus cardinal 5 denotes the set of my fingers and thumb. Cardinal 5 is bigger than the set of my fingers alone, 4, and includes it. Addition can be defined in these terms fairly straightforwardly in terms of the cardinality of the union of two disjoint sets, e.g. each of my hands has the cardinality of the set of fingers 4, and their union has the cardinality 8. I am going to refer to the cognitive conception of cardinality as numerosity.

Ordinal numbers are defined in terms of a successor operation. 5 is the successor of 4; if you take the fourth successor of 4 you get ordinal 8. This is ordinal addition. We use ordinals in a different way, for example to number pages in a book. Page five follows page four, but does not include it, and is not bigger than it. Learning to count is in part learning to align a sequence of elements which have a fixed order, the counting words, with numerosities, so that you come to realise that when you have counted a set of objects, the last word in the count is the name of the numerosity of the set. Being able to recite the list of count words and to be able to align them in one-to-one correspondence with members of the set to be counted is not the same as having a sense of the numerosity of the set. Studies of learning to count, especially by Fuson and colleagues, and Gelman and colleagues, show how these two aspects develop at first separately, and then are integrated as the child understands more. Thus, it takes months or years for the child who knows the sequence of counting words and can put each word into one-to-one correspondence with the objects in a set, to understand that the numerosity of a set can be established by counting. Fuson suggests that children may notice that when they count a set 'one two three', they get the same number as when they 'subitize' the set – that is, recognise its numerosity without counting. Most of us can recognise numerosities up to about 4 without explicit counting, and this may be an important pre-verbal capacity. This helps the child realise that counting up to N is a way of establishing that a set has N objects in it. Repeating the count, and getting the same as the number obtained from subitizing, will reinforce the idea that every number name represents a unique numerosity.

Now, one can match a definite sequence of words to members of a set without using number words at all, for example by matching each member with a letter of the alphabet or a month of the year. Of course, if you already know that this procedure will yield a numerosity, then you will know that 'f' objects or 'June' objects will represent a particular number which can be established by matching the sequence of letters or months to the number words 'one, two, three, four, five, six'. But to do this requires prior understanding of numerosities and how counting can yield them.

Counting is the basis of arithmetic for most children. Since the result of adding two numerosities is equivalent to counting the union of two disjoint sets with those numerosities, children can learn about adding by putting two sets together and counting the members of their union.

Children make use of their counting skills in the early stages of learning arithmetic. The number words, as we noted in the introduction, have both a sequence and a numerosity (or cardinal) meaning. As Fuson and Kwon point out, 'in order for number words to be used for addition and subtraction, they must take on cardinal meanings.' Children often represent the numerosity of the addenda by using countable objects, especially fingers, to help them think about and solve arithmetical problems. Thus, both developmentally and logically, arithmetic depends on numerosities.

Dyscalculia as a deficit in the concept of numerosity

Our hypothesis is that dyscalculia is a deficit in the grasp of numerosities. Dyscalculics do not have an intuitive idea of threeness or sevenness – though may know that sevenness can be the result of counting. Without these concepts, acquiring arithmetic will be like learning a poem in a language you do not know. It's possible, but you do not understand the words, or the grammar, and you will not recognise paraphrases or be able to spot mistakes.

Dyscalculic children, and adults, will be much worse at arithmetic than their peers, and the usual diagnostic procedure is to set a criterion for how much worse on a test of arithmetical skills. Different authors use different criteria, but the typical criteria are two standard deviations worse than the control or two years behind the age cohort. Other authors, including Geary, use a much laxer criterion for the inclusion in a group with 'math difficulties' or 'arithmetic learning difficulties' which can be the bottom 25% or even 35% of the cohort.

However, tests of arithmetic rely on how well educated you are, are there will be many reasons for being bad at arithmetic besides dyscalculia. Our approach has been to use tests of numerosity estimation and numerosity ordering as a measure of the *capacity* to learn arithmetic, which can be almost completely independent of education. The tests of estimation and ordering are shown in Figure 1.

On this approach, the child can be poor at arithmetic and dyscalculic, poor at arithmetic and not dyscalculic, but, in the theory, cannot be dyscalculic and good at arithmetic. Karen Landerl, Anna Bevan and I used this approach in a small study for the DfES. Our aim was to see whether low scores on these capacity tasks were indeed the signatures of dyscalculia

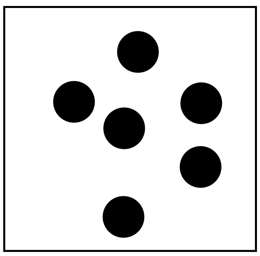
In the part of the study I will report here, there were 10 dyscalculic 9-year-olds and 18 matched controls. We wanted to ensure that our sample were really poor at maths, and so took the lowest 5% of age-group on timed arithmetic, who were also identified by teachers as having particular problems with learning arithmetic. In order to rule out complicating factors, children in the dyscalculia group had normal or superior IQ, and also normal reading, language, short-term memory. This is not to say that a child cannot have dyscalculia, dyslexia, low IQ, poor language and poor memory all at once, but our aim was to establish that one could

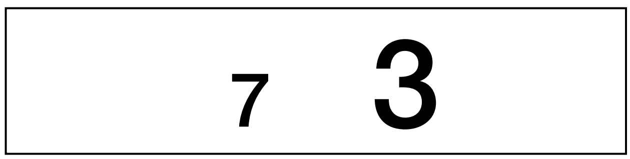
Figure 1. Tests of estimation and ordering

Panel 1A – below left, shows a standard test of estimating numerosities. Typically there are 1 to 9 dots randomly arrayed and the subject had to respond as quickly as possible with the number name (in this case 'six'). Response time (RT) is related to the number of dots: for 1 to 4 dots, the RT increases at about 60 ms per dot, for 5 to 9 dots it increases by about 200 ms per dot. This difference has been interpreted as suggesting that two processes are at work.

Panel 1B – below right, shows a standard test for ordering numerosities. The subject has to name the numerically larger number (here 7). In a second task, the subject has to name the taller (physically larger) number (here 3). This enabled us to assess both the time taken to make numerical judgements and physical judgements.

Both tasks have been adapted for use by teachers and educational psychologists, by asking for button press judgements instead of spoken responses. For estimation, the subject compares the dots with a numeral and presses a button designated as 'match' or 'non-match'; for ordering, the subject presses a button under the larger number (or taller number in the physical size task).





be dyscalculic without any of these other cognitive disabilities

On formal testing, the dyscalculic children were much slower and less accurate than their peers on single-digit addition, subtraction and multiplication; this is not surprising, since they were selected for being poor at arithmetic. They were also strikingly slower at our tasks of numerosity estimation and ordering.

To give you some idea of an individual child affected in this way, JB is not untypical. When we saw him he was 9 years 7 months old, right handed, not dyslexic and normal in all school subjects except maths, which he found impossible. On formal testing, he failed even the easiest arithmetic questions of the British Abilities Scale. Nevertheless, he was able to read and write numerals, and could count up to 20, albeit slowly. Using his counting ability he could answer, 'what comes next?' questions, such as what comes next after 3, which meant that he had a good sense of the sequence number words. However, he believed that 3+1 is 5, and in general was exceptionally poor on the simplest arithmetic tasks in out battery. On the tests of numerosity estimation, he accurately estimated dots up to 3, but guessed when there were four or more dots. He seemed to find numerosity ordering impossible. So here was an intelligent, sociable boy, who did well at school, but whose grasp of basic numerical concepts was disastrous.

The idea that grasp of basic numerical concepts is at the root of dyscalculia is usefully captured in the DfES definition: 'A condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may *have difficulty understanding simple number concepts, lack an intuitive grasp of numbers*, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.' (DfES, 2001, emphasis added.)

Since slow and inaccurate performance on our tests of capacity seemed to be good predictors of severe arithmetical learning difficulty, we have developed a classroom version that teachers and educational psychologists can easily use.

Our methods have now been used in the largest prevalence study so far undertaken. Vivian Reigosa and her colleagues have screened a cohort of over 11,000 children from 6 to 14 years in the Centro district of Havana. They found that 6.7% were dyscalculic on the basis of numerosity estimation and ordering tests, and this predicted poor performance on standardized tests of school mathematics. This is very much in line with the previous best estimate, from a cohort study in Tel Aviv, which found a prevalence of 6.4% using a criterion of two years behind the mean for the age group.

Dyscalculia also seems to be persistent in at least some cases. We have now seen many severely affected adults, but we do not know what proportion of childhood dyscalculics is still dyscalculic in adulthood. There have been no longitudinal studies of the effects of intervention specific to dyscalculia. In fact, there are very few interventions targeted specifically at dyscalculia. We have recently proposed a teaching scheme which focuses on what I have argued is the key deficit, and is therefore aimed at helping children understand basic numerical concepts. Dorian Yeo, the co-author, and a highly experienced special needs teacher, has had good early results in pilot studies, but the whole programme needs to be properly evaluated with different teachers and children with a range of diagnosed number problems.

We can now carry out a reliable differential diagnosis, separating the true dyscalculics with a deficit in basic number concepts from those of us who are bad at maths for other reasons. We are beginning to understand the brain systems for numerical processing, and how they can go wrong. And now we are in a position to scan these brains to look for abnormal activation, and to begin to look for a genetic basis of the condition.

However, compared with dyslexia, research is at a very early stage. We will need to do basic things like document the types and degrees of dyscalculia, explore patterns of hereditability and trace the development of normal and abnormal brain systems for number processing. Apart from Dorian Yeo's groundbreaking work, there is very little in the way of intervention methods targeted at diagnosed dyscalculia. Even here, the intervention programme has not yet been fully evaluated. Because everyone counts, it is vital that those who do not understand numbers are identified as early as possible, supported, just as dyslexics are supported within the school system, with specialised teaching, and properly-trained learning support assistants.

The full text of the lecture will be published in the *Proceedings of the British Academy*.