DAWES HICKS LECTURE ON PHILOSOPHY

# TERMS AND SENTENCES: THEOPHRASTUS ON HYPOTHETICAL SYLLOGISMS

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## I

FORMAL logic has a double pedigree. One tradition supposes that the inferences which logic studies are grounded upon terms and upon certain relations holding among terms. The fundamental logical notion is that of predication, a relation which associates two items of the same logical category. At the heart of every proposition lies the form 'S is P', where S and P are terms and the copula 'is' makes the predicative association. Propositions are then differentiated according to quantity (universal or particular), quality (affirmative or negative), and modality (assertoric, apodeictic, problematic). The paradigm inference is an assertoric syllogism in *Barbara*, the schema for which runs thus:

$$\frac{\begin{array}{l}\text{Every S is M}\\ \text{Every M is P}\end{array}}{\text{Every S is P.}}$$

The tradition for which *Barbara* is the paradigm may be called *term logic.*

A second tradition discovers the basis of all inferences in sentences or sentential functions and in certain relations holding among sentences. The fundamental notion is that of sentential connection.[1] Propositions, in this tradition, do not lack internal

[1] See M. Dummett, *Frege: Philosophy of Language* (London, 1973), p. 21: 'The expressions which go to make up atomic sentences—proper names (individual constants), primitive predicates and relational expressions—form one type: sentence-forming operators such as sentential operators and quantifiers which induce reiterable transformations which lead from atomic to complex sentences form the other. . . . Logic properly so called may be thought of as concerned only with words and expressions of the second type.'

articulation; indeed this tradition too uses a concept of predication. But here predication is not a relation among items of the same category but a tie which associates items of different categories. Sentences which express predicative propositions conjoin items of different linguistic categories, and the difference is brought out by the standard symbolic notation. In formulae such as 'Fa' or 'Rab' the difference between upper- and lower-case letters reflects an underlying categorial difference. The paragon inference in this tradition is *modus ponendo ponens*, the schema for which is:

If P, then Q
P
———————
Q

The tradition for which *modus ponens* is the paragon I shall call *sentence logic*.[1]

The two traditions are rivals.[2] Each offers a distinct philosophy of logic, each presents a distinct understanding of the fundamental nature of the subject, each intends to develop a complete and exhaustive system of inference. There is a temptation to regard the two traditions not as rivals but as partners. Standard modern treatments of formal logic divide the subject into two main parts, the propositional calculus and the predicate calculus. Then may we not regard term logic as an essay in the predicate calculus, sentence logic as an essay in the propositional calculus? In that case, the two traditions, whatever their historical rivalry, are best construed as offering complementary treatments of the two main parts of logic.

That irenic suggestion is to be resisted. *Barbara*, the paradigm inference of term logic, doubtless belongs in the predicate calculus;

[1] The label is not ideal, if only because the phrase 'sentence logic' may be confused with 'sentential calculus'.

[2] The rivalry is pervasive, but it perhaps shows most clearly at the basic level. According to term logic, *every* proposition has a kernel which is syntactically symmetrical: if 'X is Y' is well-formed, then so too is 'Y is X'. According to sentence logic, the fundamental form of the proposition is syntactically asymmetrical: if 'X(Y)' is well-formed, then 'Y(X)' is ill-formed. It is a further question how serious such syntactical disagreements are: they might be thought to reflect deep semantic disagreements and hence to mark a philosophical or metaphysical divide; they might be thought, at the other extreme, to amount to little more than squabbles about notation. I return briefly to this question at the end of the paper. (Note that even were the disagreements merely notational, the two traditions would remain rivals: the rivalry would perhaps be trifling or sham, but a sham rivalry is not the same thing as a real partnership.)

but term logic presents—or hopes to present—a treatment of propositional inferences as well as of predicate inferences. *Modus ponens*, the paragon inference of sentence logic, belongs in the propositional calculus, but sentence logic embraces predicate as well as propositional inferences. The two traditions make rival claims to the whole domain of logic.

## II

The modern hero of sentence logic is Gottlob Frege. Since Frege's time, indeed, formal logic has been dominated by the sentence tradition, so that our current textbooks are almost all written within that tradition. Every tiro quickly learns how to reconstrue *Barbara* within the framework of sentence logic, and it is frequently supposed that a Fregean sentence logic, or at least a logic based on Frege's system, is the only viable kind of formal logic.

The modern hero of term logic is Leibniz. In logic Leibniz was less original than Frege, and he never produced the systematic treatment of logic which he had planned. But his numerous scattered thoughts can be collected into a coherent presentation of term logic.[1] Moreover, a neo-Leibnizian system has recently been elaborated in technical detail and with considerable sophistication.[2] Sentence logicians may refer to term logic as a dead Titan, but a requiem would be premature.

## III

Term logic was not born with Leibniz. It was two thousand years old when Leibniz wrote, and its original inventor was Aristotle. Although Aristotle's syllogistic underwent various transformations and developments at the hands of his ancient and medieval followers, we may fairly name the theory championed by Leibniz Aristotelian or Peripatetic logic. Equally, Frege did not invent sentence logic. He too had Greek predecessors. Sentence logic was first developed by the Stoic logicians, of whom Chrysippus was the

[1] See especially H.-N. Castañeda, 'Leibniz's Syllogistico-Propositional Calculus', *NDJFL* 17 (1976), 481–500; cf. F. Sommers, 'Frege or Leibniz', in vol. iii of M. Schirn (ed.), *Studies on Frege* (Stuttgart, 1976) [= 'Leibniz's Programme for the Development of Logic', in R. S. Cohen, P. K. Feyerabend, and M. Wartofsky (eds.), *Essays in Memory of Imre Lakatos* (Dordrecht, 1976)]; H. Ishiguro, 'Leibniz on Hypothetical Truths', in M. Hooker (ed.), *Leibniz—Critical and Interpretive Essays* (Manchester, 1982).

[2] F. Sommers, *The Logic of Natural Language* (Oxford, 1982).

chief, and the theory which Frege championed may reasonably be called Stoic or Chrysippean logic.[1]

In antiquity, Peripatetic and Stoic logic, Aristotle and Chrysippus, were usually treated as rivals. Each system had its supporters who debated with—and sometimes slanged—one another.[2] Our knowledge of the dispute is partial in both senses of the word: the evidence we possess is fragmentary, and it derives mainly from the ancient commentaries on Aristotle's *Organon*. But it is clear that the dispute was long drawn out, that it involved a number of difficult issues in logical theory, and that it was conducted—on occasion at least—with a remarkable subtlety.

Modern scholars sometimes speak as though Aristotle invented the predicate calculus, Chrysippus the propositional: each having elaborated one part of logic, they should stand together as the twin inventors of the two complementary halves of standard modern logic. If that is right, then the ancient dispute was fatuous, each party ignorantly taking a fragment of logic for the whole. No doubt there is some truth in that idea; for the dispute was a complex one, and in part, like any other philosophical dispute, it surely turned on confusions and misapprehensions. But that is not all there was to the matter. It is more plausible historically, and more interesting philosophically, to regard the dispute as, at bottom, a confrontation between term logic and sentence logic. The disputants were partisans of different ideologies, and their disagreement reflected more than a trifling misconception of their own and each other's views: it manifested a genuine puzzlement about the underlying nature of logical inference.

## IV

Term logicians hold that all formally valid inferences can be represented within the framework of term logic. The old Peripatetic logicians were committed to a somewhat stronger thesis; for they held that all formally valid inferences could be construed within an extension of Aristotelian syllogistic. The task of establishing that thesis constitutes what I shall call the Peripatetic programme.

The Peripatetic programme, as the ancient Peripatetics were

[1] Leibniz was, of course, familiar with his Greek predecessors, whereas Frege (to the best of my knowledge) was not. Where Leibniz developed, Frege reinvented. Nevertheless, we may properly set Frege in the Stoic tradition.

[2] On the dispute see especially I. Mueller, 'Stoic and Peripatetic Logic', *AGPh* 51 (1969), 173-87; M. Frede, 'Stoic vs. Aristotelian Syllogistic', *AGPh* 56 (1974), 1-32.

aware, faced three main objections.[1] First, the terms of Aristotelian syllogisms are all *general*. But some inferences involve propositions which contain *singular* terms (proper names, for example). How can term logic deal with such inferences? Secondly, the terms of Aristotelian syllogisms correspond to *monadic* or one-place predicates. But some inferences appear to turn essentially upon the presence of *polyadic* or relational predicates. How can term logic incorporate such relational inferences? Thirdly, the propositions in Aristotelian syllogisms are all simple or *categorical*. But some inferences rely upon *complex* propositions (conditionals, disjunctions, conjunctions, etc.) and appear to depend upon the particular character of the complexity. How can term logic encompass complex propositions?

## V

The later Peripatetics customarily stated the third difficulty, which concerns complex propositions, in the following form: How can hypothetical syllogisms be reduced to categorical syllogisms?

A syllogism is categorical if each of its constituent parts—its premisses and its conclusion—is categorical.[2] Categorical propositions are simple propositions, and a proposition is simple provided that it does not contain two or more propositions as components.[3] In practice, the Peripatetics held that every

[1] Leibniz encountered the same difficulties: for a succinct account see B. Russell, *The Philosophy of Leibniz* (London, 1900), pp. 12–15; see also G. Engelbretsen, *Three Logicians* (Assen, 1981), and the essays referred to on p. 281 n. 1.

[2] The word *κατηγορικός* is usually translated—or transliterated—as 'categorical'. It should properly be translated as 'predicative': the Peripatetics called simple propositions 'predicative propositions' because they all supposedly exhibit the predicative *τὶ κατὰ τινός* structure. But the word 'categorical' is so well entrenched in modern discussions of term logic that it would be futile to insist upon the correct translation.

[3] Here I conflate the Peripatetic and the Stoic terminologies. My use of the word 'categorical' is Peripatetic. (The Stoics used *κατηγορικός* in a special sense of their own.) The distinction between simple (*ἁπλοῦς*) and non-simple is Stoic (e.g. Diogenes Laertius, vii. 68; Sextus, *M* viii. 93), and so too is the account of simplicity I give in the text. But the Peripatetics called their categorical propositions 'simple' (e.g. Alexander, *in A.Pr.* 11. 17–18), and their distinction between categorical and hypothetical propositions can—for present purposes—be assimilated to the Stoic distinction. Note that the *negations* of simple propositions are themselves simple; for if 'P' is simple, then 'not-P' cannot contain two or more propositions as components. (What of 'not-not-P'? Is that, too, simple, or does it contain both 'P' and 'not-P' as components? It is simple; for a component of a component of X is not itself a component of X. Similarly,

(assertoric) categorical proposition can be construed as having one of the four standard Aristotelian forms: 'Every S is P', 'Some S is P', 'No S is P', 'Some S is not P'.

A syllogism is hypothetical if at least one of its constituent parts is hypothetical. A proposition is hypothetical if it contains at least two propositions as components. To us the word 'hypothetical' suggests the notion of conditionality. Conditional propositions of the form 'If P, then Q' are indeed hypothetical; but so too are disjunctions, conjunctions, and—in principle, at least—propositions compounded by such connectives as 'since', 'because', 'in so far as', etc.

In practice, Stoic logic, in its classical form, limited its attention to three sentential connectives: 'if', 'or', 'and'.[1] And the Peripatetics normally divide hypothetical propositions into just two types, conditionals and disjunctions.[2] I shall not discuss the reasons for these limitations; for the hypothetical propositions with which I shall be concerned are in fact all conditional in structure.

## VI

The later Peripatetics observed with regret that Aristotle had said very little about hypothetical syllogisms.[3] In two difficult

propositions such as 'Chrysippus believed that Heraclitus held that the world was periodically consumed by fire' are presumably also simple. But I do not know of any ancient text that discusses such complicated simple propositions.)

[1] Stoic logic also uses the sentential operator 'not' or 'it is not the case that'; but unlike modern logicians the Stoics did not treat 'not' as a sentential *connective* (see above, p. 283 n. 3).

[2] See, for example, Alexander, *in A.Pr.* 11. 20; Galen, *inst. log.* iii. 1–5; Apuleius, *de int.* 177. 3–10; 'Alcinous', *didasc.* vi (p. 158 H.); Boethius, *hyp. syll.* I. i. 5, iii. 2 (cf. M. W. Sullivan, *Apuleian Logic* (Amsterdam, 1967), pp. 24–30). The issues here are intricate. The Peripatetics did not merely overlook the various connectives which the Stoic logicians recognized. Crudely speaking, the Stoics adopted a linguistic approach, recognizing as many compound propositions as there were distinct sentential connectives (but they used a loaded notion of distinctness). The Peripatetics regarded such an approach as superficial: they held that there were only two forms of hypothetical propositions because there were only two ways in which facts could in reality be connected. The dispute is again philosophical or 'ideological': that is to say, it reflects different underlying conceptions of the nature of logic.

[3] Alexander, *in A.Pr.* 389. 31–390.1; Philoponus, *in A.Pr.* 242. 14–15, 359. 30–2; [Ammonius], *in A.Pr.* 67. 35. Later authors were not always so disappointed. John of Salisbury's comment is amusing enough to bear transcription: *sed forte ab Aristotile de industria relictus est hic labor, eo quod plus difficultatis quam utilitatis videtur habere liber illius qui diligentissime scripsit* [i.e.

chapters of the *Prior Analytics*, *A* 23 and *A* 44, he passes some remarks on 'syllogisms from a hypothesis'.[1] At the end of *A* 44 he promises that 'we shall discuss later the varieties of arguments of this kind and the number of ways in which things are said to be "from a hypothesis"' ($50^a40-^b2$). He did not keep his promise. Moreover, he added that 'for the present let this much be clear to us—that it is not possible to analyse such syllogisms into the figures [of categorical syllogistic]' ($50^b2-3$). Advocates of the Peripatetic programme were disappointed that Aristotle failed to keep his promise, and they must have been depressed by his magisterial assertion that hypothetical syllogisms could not be 'analysed into' the categorical figures.

If Aristotle was discouraging, his immediate successors lightened the gloom. According to Alexander of Aphrodisias, 'Theophrastus mentions ⟨hypothetical syllogisms⟩ in his own *Analytics*, and so do Eudemus and certain others of Aristotle's associates' (*in A.Pr.* 390. 2–3).[2] Later, Philoponus and Boethius repeat the assertion, in slightly different forms.[3]

These reports have occasioned much comment. It is uncertain who the 'other associates' of Aristotle were: apart from Theophrastus and Eudemus, the early Peripatetics do not appear to have had much taste for logic. It is disputed exactly what sorts of hypothetical syllogisms Theophrastus and the rest discussed. It is not clear how extensively the early Peripatetics treated the subject—Philoponus speaks of 'long treatises' while Boethius appears to imply that the discussions were superficial and brief. These general questions are of some importance to the history of formal logic; but they may be ignored here.[4] For on one aspect of the matter we are in fact tolerably well informed.

Boethius]. *profecto si hunc Aristotiles more suo exequeretur, verisimile est tante difficultatis fore librum ut preter Sibillam intelligat nemo* (*Metalogicon*, iv. 21).

[1] See most recently G. Striker, 'Aristoteles über Syllogismen "aufgrund einer Hypothese"', *H* 107 (1979), 33–50; J. Lear, *Aristotle and Logical Theory* (Cambridge, 1980), ch. 3. Aristotle's 'syllogisms from a hypothesis' should perhaps *not* be construed as hypothetical syllogisms, i.e. as arguments from hypothetical premisses. Aristotle's successors, however, certainly discussed hypothetical syllogisms, and they certainly supposed that Aristotle himself had undertaken to discuss them.

[2] = Theophrastus, F 29 Graeser, frag. 33c Repici.

[3] Philoponus, *in A.Pr.* 242. 18–21 (= Theophrastus, F 29 G, frag. 33b R); Boethius, *hyp. syll.* I. i. 3 (= Theophrastus, F 29 G, frag. 33a R).

[4] See J. Barnes, 'Theophrastus on Hypothetical Syllogisms', in the Moraux Festschrift (Berlin, forthcoming).

## VII

One of the topics which Theophrastus certainly treated was that of 'wholly hypothetical' syllogisms. Our main source for that treatment is a long passage in Alexander's commentary on Aristotle's *Prior Analytics* in which he draws on the first book of Theophrastus' *Prior Analytics*.[1] The passage is not a quotation from Theophrastus, nor indeed does it purport simply to document Theophrastus' views. None the less, we can extract his views from Alexander's commentary with reasonable confidence.[2]

The passage is formally devoted to an examination of *A.Pr.* $45^{b}19$ ('we must investigate and distinguish the number of ways in which arguments from a hypothesis . . .'). Alexander's question is whether or not *all* hypothetical syllogisms can be 'reduced'[3] to

[1] *In A.Pr.* 325. 31–328. 7: text and translation in the Appendix, below. Most of the passage appears as Theophrastus, F 30 G, frags. 34a and 34b R. See also Philoponus, *in A.Pr.* 302. 6–23, which is, however, a highly condensed and somewhat inaccurate version of Alexander's account.

[2] L. Obertello, *A. M. Severino Boezio: de hypotheticis syllogismis* (Brescia, 1969), p. 45, states that 'The account closest to the thought of Theophrastus seems to be that of Philoponus, whereas Alexander presents his own personal reworking of it'. Obertello gives no reason for that curious judgement.

[3] 'Reduce' translates *ἀνάγειν*, for which *ἀναλύειν* is a virtual synonym; 'reduction' is *ἀναγωγή*, which is interchangeable with *ἀνάλυϲιϲ*. Those terms are technical, or at least semi-technical, in Peripatetic logic, but they are not given any formal definition. Aristotle uses *ἀνάγειν* in two main contexts. (1) He speaks of 'reducing' one syllogism to another (e.g. *A.Pr.* $29^{b}1$, the reduction to *Barbara* and *Celarent* of the other valid moods). The origin of this sense is to be found in the notion of *ἀνάγειν εἰϲ ἀρχάϲ*, 'reducing to first principles'. It corresponds closely to the notion of derivation in modern logic: an argument schema S is 'reduced' to a set of schemata S* just in case S is derivable from S*. (2) Aristotle also speaks of 'reducing' arguments 'into the figures' (e.g. *A.Pr.* $46^{b}40$). Here the corresponding modern notion is that of formalization. An argument (expressed in natural language) is 'reduced' to a schema S just in case it is formalized as an instance of S.

When the Peripatetics speak of 'reducing' hypothetical syllogisms, it is not always clear which usage, (1) or (2), they have in mind. And in fact it does not matter. For to 'reduce' *modus ponens*, in sense (1), to a set of categorical schemata is equivalent to 'reducing', in sense (2), to categorical form any ordinary language argument which, so to speak, invites formalization by way of *modus ponens*.

I have spoken of the Peripatetic programme as involving the 'reduction' of hypothetical syllogisms to categorical syllogisms. There is a hitch here. Take *modus ponens*, a 'mixed' hypothetical syllogism. Alexander does *not* maintain that *modus ponens* inferences can be represented within categorical syllogistic (e.g. *in A.Pr.* 386. 5–8, following *A.Pr.* $50^{a}16$). Rather, he argues as follows. The categorical premiss of a *modus ponens* inference either is or is not the conclusion of a further inference. If it is the conclusion of a further inference, then that categorical syllogisms by what he has called 'the method of selection'.[1] A particular problem is caused by one sort of

inference is either a categorical syllogism or a mixed hypothetical syllogism. If it is a categorical syllogism, then the original *modus ponens* inference is shown to rest upon, and hence to be 'reduced' to, a categorical syllogism (see, for example, *in A.Pr.* 263. 15–17). If the second inference is hypothetical, then *its* categorical premiss either is or is not the conclusion of a further inference—and the earlier argument is repeated (see *in A.Pr.* 387. 5–11; cf. Philoponus, *in A.Pr.* 241. 31–242. 13). If, finally, the categorical premiss of a *modus ponens* argument is not the conclusion of a further inference (because it is 'evident', ἐναργής), then the *modus ponens* inference is not a cυλλογιcμόc at all (*in A.Pr.* 263. 7–11, 265. 5–10, 388. 12–17).

In all that, Alexander is probably following Theophrastus (see *in A.Pr.* 388. 17–20). As Alexander presents the argument, it is badly confused. Most obviously, it cannot count as a *reduction* of *modus ponens* to categorical syllogistic. Three views of Alexander's procedure are possible. (A) Alexander believes, mistakenly, that the procedure *does* 'reduce' *modus ponens*, in sense (1), to categorical syllogistic. (B) Alexander realizes that he cannot produce a genuine reduction but thinks that his procedure is the nearest he can get to one. (C) Alexander is operating with a new notion of 'reduction': he thinks, perhaps, that his procedure shows that categorical syllogisms are in a sense *prior* to hypothetical syllogisms, and that that notion of priority can properly ground a new notion of reducibility.

If we adopt (C), then we must infer that the Peripatetic programme is not, after all, one of reducing all formal inference to Aristotelian term logic. That is, perhaps, the most charitable way to construe Alexander's procedure. But I am inclined to think that charity is misplaced here. Alexander himself does not clearly state what he has in mind by a reduction: I suspect that, if pressed, he would acknowledge that he is really after a genuine sense (1) derivation. In that case, view (A) is correct and Alexander is in a muddle. (Something similar is certainly true of Alexander's treatment of relational inferences).

[1] Alexander is referring to the notion of 'selection' or ἐκλογή which Aristotle employs in *A.Pr.* *A* 27–9 (ἐκλέγειν, $43^b11$, etc.; ἐκλογή, $44^b26$ etc.). For an example of the 'method of selection' see *in A.Pr.* 324. 5–15. Alexander considers the mixed hypothetical syllogism:

If the soul is always in motion, the soul is immortal
The soul is always in motion
———
The soul is immortal.

'We shall take the terms in the μετάληψιc or πρόcληψιc [i.e. the second, categorical, premiss]—they are *soul* and *always in motion*—and make the selection with regard to them' (324. 10–12). The 'selection' picks on the term *self-moved*, and thus generates a categorical syllogism in *Barbara*.

Every soul is self-moved
Everything self-moved is always in motion
———
Every soul is always in motion.

By the 'selection' we produce a categorical syllogism which establishes one of hypothetical syllogism; 'for wholly hypothetical arguments . . . will be thought not to be amenable to proof by selection' (326. 8–10).

What does the phrase 'wholly hypothetical' mean? Alexander does not explain it, but Philoponus says that wholly hypothetical syllogisms are so called because 'all the propositions assumed are hypothetical' (*in A.Pr.* 243. 16; cf. 302. 9–12).[1] They thus contrast with, say, *modus ponens* arguments, in which only one of the assumed propositions is hypothetical. (Arguments of that sort were customarily called 'mixed' hypothetical syllogisms.) According to Philoponus (302. 9), Theophrastus himself used the phrase 'wholly hypothetical syllogism'. But Alexander reports that Theophrastus called such syllogisms 'arguments in virtue of an analogy' (*in A.Pr.* 326. 9) on the grounds that 'the premisses are analogous to each other and the conclusion to the premisses—they all show a similarity' (326. 11–12). But that is at best a strained explanation, and we may well wonder if Alexander has not misunderstood Theophrastus' text.[2]

Philoponus adds that wholly hypothetical syllogisms are also called 'hypotheticals with three components' 'because these syllogisms are inferred by way of at least three propositions' (*in A.Pr.* 243. 16–17).[3] Alexander, however, appears to treat 'hypotheticals with three components' as a special *kind* of wholly hypothetical syllogism (*in A.Pr.* 326. 9), and he is right to do so.

In principle, then, a wholly hypothetical syllogism is an argument in which each premiss, and also the conclusion, is hypothetical or contains at least two propositions as components. In fact, we shall find that Theophrastus concerned himself with only one special sort of wholly hypothetical syllogism.

the premisses of the hypothetical syllogism, and the hypothetical syllogism is thus 'reduced' to a categorical syllogism (see above, p. 286 n. 3).

[1] Philoponus actually uses δι' ὅλου rather than δι' ὅλων (so too the *documentum Ammonianum*: [Ammonius], *in A.Pr.* xi. 2).

[2] It is tempting to guess that Theophrastus wrote: λέγω δὲ αὐτοὺϲ ϲυλλογιϲμοὺϲ κατὰ ἀναλογίαν. He meant: 'It is in virtue of an analogy [i.e. the analogy with categorical syllogisms] that I call them "syllogisms"'. Alexander wrongly took his ambiguous sentence to mean: 'I call them "syllogisms in virtue of an analogy"'—and he then concocted an implausible explanation for the nomenclature. That conjecture has the merit of associating the ἀναλογία at *in A.Pr.* 326. 9 with the ἀναλογίαι later in the passage. It has the demerit of imputing a surprising carelessness to Alexander.

[3] Cf., for example, Alexander, *in A.Pr.* 265. 16, 390. 19; [Ammonius], *in A.Pr.* xi. 1; Boethius, *hyp. syll.* I. vi. 2.

## VIII

Alexander's first illustration of a wholly hypothetical syllogism is this:

an argument of the following sort is wholly hypothetical:

If A, then B
If B, then C
---
If A, then C.

(Here the conclusion too is hypothetical.) E.g.:

If he is a man, he is an animal
If he is a animal, he is a substance
---
If he is a man, he is a substance

(*in A.Pr.* 326. 22–5).

It is worth noting, in passing, that the *argument*—the wholly hypothetical *syllogism*—is contained in the last three lines of that extract. The lines using the letters 'A', 'B', 'C' do not present a wholly hypothetical syllogism because they do not present a syllogism at all: their function is to exhibit the skeleton or underlying structure of wholly hypothetical syllogisms.

It is also worth noting that there is no warrant for introducing material implications into Alexander's syllogism. Sophisticated scholars regularly replace Alexander's 'if . . . then ——' by the horseshoe of material implication. Now the Greeks knew something rather like material implication, and certain ancient arguments invite formulation by way of the modern connective. But the analysis of conditional propositions was hotly disputed among the ancient philosophers. Material implication was not the orthodox, let alone the only, variety of conditional which they recognized, and it is seriously misleading to throw horseshoes casually into Greek texts. The ordinary Greek εἰ should be translated by the ordinary English 'if'; and unless there is some indication that a conditional should be given some particular technical analysis, we must be content with an informal and non-technical understanding.[1]

[1] Did the Peripatetics have an official or orthodox account of the truth-conditions of conditional sentences? (i) Theophrastus said *something* about the meaning of εἰ—at any rate, in his *A.Pr.* he distinguished εἰ from ἐπεί (see Simplicius, *in Cat.* 552. 31–553. 4). (ii) Some scholars ascribe to the Peripatetics the account of conditionals in terms of ἔμφαϲιϲ which Sextus preserves (*PH* ii. 110–12; cf. Galen, *meth. med.* x 126K). But the ascription is dubious (cf. M. Frede, *Die stoische Logik* (Göttingen, 1974), pp. 90–3). (iii) Alexander may have held a special view about the nature of conditional

## IX

Alexander's schema raises one difficult question. What is the syntactic status of the letters 'A', 'B', 'C'? The grammar of 326. 23 demands that they be *sentential* letters: like the 'P' and 'Q' of modern propositional calculus, they hold the place of complete sentences. No other replacement of 'A' and 'B' will make grammatical sense of the schema 'If A, then B'. And it seems that Alexander does as a matter of fact replace 'A' and 'B' by complete sentences, namely by 'he is a man' and 'he is an animal'.

Yet most scholars hold that Alexander's 'A', 'B', 'C' are *term* letters, having the status which they regularly have in Aristotle's *Analytics*, where they hold the place of general terms like 'man', 'animal', 'substance'.[1] Now if 'A' and 'B' are really meant as term letters, we must plainly treat 'If A, then B'—'If man, then animal'—as elliptical. And there are more ways than one of understanding the ellipsis.

Philoponus, in his account of hypothetical syllogistic, perhaps intends 'A', 'B', 'C' as term letters,[2] and if that is so he probably means us to construe 'If A, then B' as 'If ⟨so-and-so is an⟩ A, then ⟨so-and-so is a⟩ B': 'If ⟨Dio is a⟩ man, then ⟨Dio is an⟩ animal'.[3] It might be thought that Philoponus' interpretation does not differ significantly from the sentential interpretation. After all, in Alexander's text (327. 12) the sentential interpreter, like Philoponus, will construe 'If man, then animal' as elliptical for 'If he is a man, then he is an animal'. But there is an important difference: whereas Philoponus takes 'man' and 'animal' as proper replacements for 'A' and 'B' and holds 'If A, then B', no less than 'If man, then animal', to be elliptical, the sentential

sentences—see below, p. 308. (iv) Parts of Boethius' logic *either* commit serious blunders *or* employ an unusual notion of the conditional. In the present state of research I do not think that we can interpret the hypothetical syllogistic of the Peripatetics with any confidence. (The same is true of Stoic logic.) But we can surely attain a partial understanding: when in the text I assert that such and such an inference is valid or invalid, I mean the assertion to hold *whatever* the proper analysis of Peripatetic conditionals should turn out to be.

[1] So, for example, I. M. Bocheński, *La Logique de Théophraste* (Fribourg, 1947), p. 114; id., *Formale Logik* (Freiburg/Munich, 1956), pp. 118–19; Obertello, op. cit. (p. 286 n. 2), p. 25.

[2] See *in A.Pr.* 302. 22, where a hypothetical conclusion is expressed schematically by the words *εἰ μὴ τὸ A ἄρα, οὐδὲ τὶ τῶν Γ*. Here 'C' can only be a term letter. But this may well betoken no more than a minor slip on Philoponus' part.

[3] See *in A.Pr.* 244. 7: *εἰ τὸ προςιὸν ἄνθρωπός ἐςτι, κτλ.*

interpreter takes 'man' to *abbreviate* the proper replacement for 'A' and does not regard 'If A, then B' as elliptical.

In his work on hypothetical syllogisms, Boethius does not explicitly offer an interpretation of earlier Peripatetic treatments. But he was writing in the Peripatetic tradition and his own way of construing 'A', 'B', 'C' can be regarded as an implicit interpretation of Theophrastus and his followers. Boethius regularly refers to 'A', 'B', 'C' as *termini*; and he regularly writes '*est* A', '*est* B', '*est* C', thus indicating beyond any doubt that he thinks that 'If A, then B' is elliptical and that 'A' and 'B' are term letters. He remarks that 'The terms of conditional propositions are usually expressed in indefinite mode, and so I have judged it unnecessary to follow out a host of propositions determinate in quantity' (*hyp. syll.* I ix. 2–3). 'If man, then animal' is 'indefinite', that is to say it is marked neither as universal nor as particular—it is unquantified. In other words, Boethius in effect regards 'If man, then animal' as an *open* sentence—'If *x* is a man, then *x* is an animal'. In practice, he usually seems to treat the open sentences as equivalent to their universal closures, thus construing 'If man, then animal' ('If A, then B') as 'If anything is a man, then it is an animal' ('If anything is an A, then it is a B').

More recently, scholars have offered yet a further interpretation. They read 'A', 'B', 'C', as term letters; they take Boethius' '*est* A', etc., as correct expansions of 'A', etc.; and they then read '*est*' existentially. Hence they gloss 'If A, then B' as 'If there are As, then there are Bs': 'If man, then animal' thus represents 'If there are men, then there are animals'.[1]

These four interpretations of 'If A, then B' imply significantly different structures for the wholly hypothetical syllogism. The first interpretation, which makes 'A', 'B', 'C' sentential, suggests that a typical wholly hypothetical syllogism might have the form:

If P, then Q
If Q, then R
―――――――
If P, then R.

The second, Philoponan, interpretation suggests rather:

If *a* is F, then *a* is G
If *a* is G, then *a* is H
―――――――――
If *a* is F, then *a* is H.

[1] So, for example, A. Graeser, *Die logischen Fragmente des Theophrast* (Berlin, 1973), pp. 98–9; L. Repici, *La logica di Teofrasto* (Bologna, 1977), p. 149.

Boethius offers a third suggestion:

> If anything is F, then it is G
> If anything is G, then it is H
> ---
> If anything is F, then it is H.

Finally, from the moderns we have:

> If there are Fs, then there are Gs
> If there are Gs, then there are Hs
> ---
> If there are Fs, then there are Hs.

The second and the fourth of these illustrative forms are special cases of the first. The third is an argument form of a somewhat different type.

The choice of interpretation is not a trifling matter. For it will determine the scope of Theophrastus' treatment of wholly hypothetical syllogisms, and hence the truth or falsity of his views and theses on the subject.

## X

The fourth, modern, interpretation can be eliminated with some confidence. Were it right, then we should have to construe the illustrative syllogism which Alexander gives at 327. 12–13 as follows:

> If there are men, there are animals
> If there are stones, there are no animals
> ---
> If there are men, there are no stones.

I doubt if anyone would naturally take Alexander's words in that way, or would readily suppose that such an argument would have struck any Peripatetic logician as an apt illustration.[1] (Apart from its inherent asininity, the argument is unlike virtually all illustrative arguments in ancient texts: ancient logicians—like their modern successors—customarily illustrate schemata by arguments which are, or are assumed to be, *sound*. In many ways that is a bad custom. But it is deeply entrenched.)

It is harder to decide among the other three interpretations. In favour of the second or the third is the fact that hypothetical syllogistic is almost invariably illustrated in our surviving texts by

[1] Cf. Boethius, *hyp. syll.* II. ix. 7, whose informal comments on a similar concrete argument make it plain that *non est lapis* means 'he is not a stone' and not 'there are no stones'. See also II. xi. 4: *si homo est, non est irrationabile; si irrationabile non est, inanimatum EUM non esse necesse est.*

examples which fit the specific argument forms which those interpretations determine. If the Peripatetics had intended the first and most general interpretation, why, it might pertinently be asked, did they always choose illustrations which were accommodated to the forms determined by the other two interpretations?

But that question has, I suspect, a perfectly trivial answer. Alexander's example at 326. 24–5, which sets the tone for the remaining illustrations, is drawn from Aristotle: it appears at *A.Pr.* $47^a$28–30. A Peripatetic logician would as a matter of course look to Aristotle for his first example of an argument form. Here that example happens to fit the second and the third interpretations as well as it fits the first. The subsequent examples, to preserve uniformity, will also have the same fit. Had Theophrastus wanted to discuss the argument forms determined by the first interpretation, he would, by virtue of his Aristotelian starting-point, have chosen as illustrative examples arguments which happen also to be accommodated to the second and third interpretations.[1]

There are at least two[2] considerations which tell in favour of the first interpretation of 'A', 'B', 'C'. First, when the Peripatetics discussed 'mixed' hypothetical syllogisms, such as *modus ponens*, they did not restrict themselves to specific forms. In other words, they discussed:

$$\begin{array}{l} \text{If P, then Q} \\ \text{P} \\ \hline \text{Q} \end{array}$$

rather than

$$\begin{array}{l} \text{If } a \text{ is F, then } a \text{ is G} \\ a \text{ is F} \\ \hline a \text{ is G} \end{array}$$

[1] Frede, op. cit. (p. 282 n. 2), p. 9, says that 'the Stoic truth-conditions for conditionals and disjunctions are so strong that examples of such propositions always seem to be true due to a certain relation between the terms, e.g. "If something is a man, he is a mortal", "If something is walking, then it is in motion". It is therefore not surprising that authors who, at least in logic, follow the Peripatetic tradition treat hypothetical arguments as if they depended on a relation between terms.' But one of the most common Stoic examples of a conditional proposition is 'If it is day, it is light', and that is *not* easily taken to exhibit the form 'If *a* is F, *a* is G'.

[2] One might also urge that the sentential interpretation is the *obvious* one, inasmuch as it does not suppose that Alexander's formal account of the structure of a wholly hypothetical syllogism is elliptical.

or

> If anything is F, it is G
> *a* is F
> ―――――――――――
> *a* is G.[1]

Given that the Peripatetics aspired to generality in the mixed cases, why should they have restricted themselves in the wholly hypothetical cases?

Secondly, there is at least *one* Peripatetic illustration of a wholly hypothetical syllogism which does *not* fit the second or the third interpretation. The so-called *documentum Ammonianum* presents the following argument as a paradigm of wholly hypothetical inference:

> If the sun is above the earth, then it is day
> If it is day, then it is light
> ―――――――――――
> If the sun is above the earth, then it is light

([Ammonius], *in A.Pr.* xi. 2–3).

Thus at least one Peripatetic logician implicitly endorses the first interpretation, construing 'A', 'B', 'C' as sentence letters.[2]

Not without hesitation I assume that the first interpretation represents Alexander's—and Theophrastus'—intention.[3] If that is right, then we may say that the wholly hypothetical arguments with which Theophrastus was concerned were of the general form:[4]

[1] Strictly speaking, this is *not* a special case of *modus ponens*; but see below, p. 313 n. 3.

[2] Boethius' illustrations of hypothetical propositions in *hyp. syll.* are usually of the special form 'If Fx, then Gx'. But they are not invariably or essentially of that form. He is happy to use the stock Stoic example, *si dies est, lux est* (e.g. I. iii. 4); his illustration of an 'accidental' hypothetical is *cum ignis calidus sit, caelum rotundum est* (I. iii. 5); and he shows what he means by a hypothetical in which *ipsius consequentiae causam positio terminorum facit* by offering the sentence *si terrae fuerit obiectus, defectio lunae consequitur* (I. iii. 7). Note also Aristotle, *A.Pr.* $57^a36$–$^b16$, where the argument is illustrated by the sentences 'If A is white, B is large', 'If A is not white, B is large', etc. Theophrastus must have been aware that not *all* conditional sentences have the form 'If Fx, then Gx'.

[3] K. Dürr, *The Propositional Logic of Boethius* (Amsterdam, 1951), p. 22, explicitly takes Boethius' *est a*, *est b*, etc., to be *propositional* variables, thus construing Boethius in the same way as I construe Theophrastus.

[4] Perhaps, to preserve full generality, we should rather write:

($H^1$) If $A_1$, then $A_2$; if $A_3$, then $A_4$; . . .; if $A_{n-1}$, then $A_n$: therefore if $B_1$, then $B_2$

where $n \geqslant 2$. For cases where $n > 4$ see below, p. 306. Our texts never consider

(H) If A, then B
If C, then D
———
If E, then F.

## XI

Theophrastus developed an analytical account of syllogisms of form (H). The first step in his analysis is the observation that 'here too [i.e. in hypothetical as well as in categorical syllogisms] there must be some middle term in virtue of which the premisses connect with one another' (326. 25–7). Philoponus repeats the point, speaking more exactly of a 'middle hypothesis' (*in A.Pr.* 243. 17–21) rather than of a 'middle term'.[1] That amounts to the metatheorem that an inference of form (H) is valid only if the two premisses have a component in common.

The metatheorem is worth stating a little more precisely. Since the component propositions of a hypothetical syllogism may be either affirmative or negative, we can reformulate (H) as:

(H*) If $\pm$A, then $\pm$B
If $\pm$C, then $\pm$D
———
If $\pm$E, then $\pm$F

—where '$\pm$X' indicates that 'X' may or may not be preceded by a negation-operator. Theophrastus' metatheorem now amounts to this: if an argument of form (H*) is valid, then either A = C or A = D or B = C or B = D. Alexander reports no argument for that metatheorem[2]—perhaps Theophrastus took it to be self-evident.[3]

Any component of the premisses which is not a 'middle'

the possibility of cases in which $n = 2$ (i.e. in which the hypothetical syllogism has a *single* premiss). Perhaps Theophrastus ruled that case out, just as Aristotle rules out categorical syllogisms with only one premiss (*A.Pr.* 40b35–6). ('But surely Theophrastus allows the validity of "If P, then Q; therefore, if not-Q, then not-P"?' Yes (see below, p. 299). And Aristotle allows the validity of 'Some A is B: therefore some B is A'. But those are 'immediate inferences', not *syllogisms*.) Chrysippus, too, rejected λόγοι μονολήμματοι (e.g. Sextus, *PH* ii. 167). See, in general, J. Barnes, 'Proof Destroyed', in *Doubt and Dogmatism*, ed. M. Schofield, M. F. Burnyeat, J. Barnes (Oxford, 1980), pp. 173–5.

[1] See also Boethius, *hyp. syll.* I. ix. 1.

[2] In categorical syllogistic Aristotle offers an argument for the corresponding metatheorem that every valid inference must contain a middle term: *A.Pr.* 40b30–41a20. Theophrastus might have translated that argument into the hypothetical mode.

[3] Theophrastus' metatheorem may seem not merely to be non-evident but actually to be false; but see below, p. 296 n. 1.

hypothesis we may call an 'extreme'. By speaking of *the* middle hypothesis, Alexander implies that each syllogism contains a *single* middle hypothesis, and hence two extremes. Theophrastus probably also assumed that the two extremes were distinct from one another; and he certainly assumed that the extremes each appeared once in the conclusion. There is no direct textual evidence for these assumptions, but they are, I think, implicit in the Peripatetic treatment of wholly hypothetical syllogistic. They are best regarded as stipulative in nature: their function is to determine a special class of syllogisms within the general schema (H*).[1]

Let us use 'M' to designate the middle hypothesis, 'E' and 'E*' to designate the extremes. And let 'IF [X, Y]' be ambiguous between 'If X, then Y' and 'If Y, then X'. Then Theophrastus' wholly hypothetical syllogisms are arguments of the specific form:

$$(\mathrm{H}^{**})\quad \begin{array}{l} \mathrm{IF}\ [\pm \mathrm{E},\ \pm \mathrm{M}] \\ \mathrm{IF}\ [\pm \mathrm{E}^*,\ \pm \mathrm{M}] \\ \hline \mathrm{IF}\ [\pm \mathrm{E},\ \pm \mathrm{E}^*]. \end{array}$$

## XII

Alexander next remarks that the 'middle term will be positioned three ways in pairings of this sort too' (326. 27–8). In categorical syllogistic, the three Aristotelian figures are determined by the 'position', or role, of the middle term:[2] the middle term is either subject in one premiss and predicate in the other, or predicate in

[1] Those assumptions exclude from the scope of wholly hypothetical syllogistic certain valid arguments of form (H*). Thus the assumption that there is just one middle hypothesis will rule out, e.g.:

If A, then B; if B, then A: therefore, if A, then B.

The assumption that the extremes are distinct will rule out, e.g.:

If A, then B; if B, then A: therefore, if A, then A.

(For similar assumptions implicit in Aristotle's categorical syllogistic see P. Thom, *The Syllogism* (Munich, 1981), pp. 27–9. There is a considerable ancient literature on the subject of 'reduplicated' propositions of the form 'If A, then A'.) The assumption that the extremes each appear in the conclusion outlaws, e.g.:

If A, then B; if A, then C: therefore, if not-C or not-B, then not-A.

It also rule out:

If A, then B; If A, then C: therefore, if A and C, then B and D

—an argument which lacks a middle hypothesis (see above, p. 295 n. 3).

[2] For the notion of 'position', θέcιc, here see G. Patzig, *Aristotle's Theory of the Syllogism* (Dordrecht, 1968), pp. 91–104.

both premisses, or subject in both premisses. The 'position' of the middle proposition in hypothetical premiss-pairings is similarly determined by its role as antecedent or consequent in the conditional premisses: it may be antecedent in one premiss and consequent in the other, or consequent in both premisses, or antecedent in both premisses. There are no other possibilities.

The three possible pairings can be set down as follows:

(P1) If $\pm$E, then $\pm$M; if $\pm$M, then $\pm$E*
(P2) If $\pm$E, then $\pm$M; if $\pm$E*, then $\pm$M
(P3) If $\pm$M, then $\pm$E; if $\pm$M, then $\pm$E*

The three pairings determine the three possible 'figures' of wholly hypothetical syllogistic: a syllogism belongs to the first figure if its premisses have the structure (P1), and so on. Just as every categorical syllogism belongs to one of the three Aristotelian figures, so every hypothetical syllogism belongs to one of the three Theophrastan figures.[1]

I have set down the three pairings in the order in which Alexander gives them. But Alexander notes that he has changed the original Theophrastan order: Alexander's second figure was Theophrastus' third, Alexander's third was Theophrastus' second (328. 2–5). Alexander tentatively undertakes to discuss the question of the ordering 'separately' (328. 6), but no such discussion survives.

Philoponus follows Alexander's order of the figures (*in A.Pr.* 302. 15–22). Boethius follows Theophrastus.[2] Neither author gives any hint that there is an alternative ordering; neither indicates that the matter is in the least controversial.

The figures of wholly hypothetical syllogistic are also mentioned in the *Didascalicus*, an introduction to Plato's thought ascribed to Alcinous by the manuscripts and to Albinus by most modern scholars.[3] The *Didascalicus* notes that Plato uses all three figures of categorical syllogistic and then remarks that 'we shall find hypothetical arguments propounded by him in many of his books—and especially in the *Parmenides*'. An illustration is produced: it belongs to the first figure, although the text does not

[1] There can be no 'fourth figure' in hypothetical syllogistic—nor is there any room for a fourth figure in Aristotelian categorical syllogistic. The history of the fourth figure is the history of a muddle: the (stipulative) determination of the figures by way of the configurations of *premiss-pairings* establishes their number as three (see Thom, op. cit. (p. 296 n. 1), pp. 24–7).

[2] See *hyp. syll.* II. ix. 2; III. i. 1, iv. 2.

[3] See C. Mazzarelli, 'L'autore del *Didaskalos*—l'Alcinoo dei manoscritti o il medioplatonico Albino?', *Rivista di filosofia neo-scolastica*, 72 (1980), 606–39.

say so. Next: 'And in the second hypothetical figure, which most people call third (in which the common term is apodosis to both extremes), he propounds the following argument. . . . And again in the third figure, called second by some, in which the common term is antecedent to both . . .' (p. 159 Hermann).[1]

The *Didascalicus* thus adverts to the hypothetical figures casually, as though they were no less familar than the categorical figures; it indicates that there was disagreement about the order of the figures—a disagreement which apparently involved several scholars; and it tacitly sides with the position favoured by Alexander.[2] Albinus was a middle Platonist, active in about AD 150. If the *Didascalicus* is his work, then we have evidence for a dispute about the hypothetical figures before the time of Alexander. (We might reasonably guess that the disputants included Boethus and Ariston, participants in the Peripatetic revival of the first century BC who showed some interest in logic.)[3] If the *Didascalicus* is by the shadowy Alcinous, we can date neither it nor the dispute to which it refers.[4]

[1] Alcinous' first two illustrations come from the *Parmenides*, his third from the *Phaedo*. The surviving ancient commentaries on *Parm* and *Pho* do not, so far as I am aware, follow Alcinous' and find wholly hypothetical syllogisms in Plato's text. (Alcinous' first example draws on *Parm* 137D: this text is analysed by Proclus, *in Parm* 1111. 1–6, as a simple *modus ponens* argument.)

[2] It is likely that more information on the dispute is contained in the Arabic commentaries on Aristotle. The hypothetical figures were certainly discussed by Avicenna and by Averroes.

[3] On Boethus and Ariston see P. Moraux, *Der Aristotelismus bei den Griechen I*, Peripatoi 5 (Berlin, 1973), pp. 164–9, 186–92.

[4] The anonymous commentary on the *Theaetetus* has the following note on *Tht.* 152 BC (I print the text with Diels-Schubart's supplements): *ὅταν γὰρ σκοπῇς, κατὰ τὸ τρίτον σχῆμα ἠρώτηται αὐτῷ ὁ λόγος· οἷα ἑκάστῳ φαίνεται, τοιαῦτα καὶ ἔστιν αὐτῷ· καὶ οἷα φαίνεται, τοιαῦτα καὶ αἰσθάνεται· ἐξ ὧν συνάγεται· οἷα ἕκαστος αἰσθάνεται, τοιαῦτα καὶ ἔστιν αὐτῷ.* Diels-Schubart, pp. xxx–xxxi, compare the *Didascalicus*, implying that the 'third figure' in anon. is the third hypothetical figure of 'Alcinous'. The precise date of anon. is uncertain, but it is indisputably earlier than Alexander: the papyrus itself is from the first half of the second century AD, and Harold Tarrant has plausibly dated the commentary to the second half of the first century BC ('The Date of Anon. *in Theaetetum*', *CQ* 33 (1983), 161–87). It is tempting to infer that a non-Theophrastan ordering of the hypothetical figures was current two centuries before Alexander. That is an exciting conclusion, but it is based on a dubious premiss; the argument exposed in anon. is not hypothetical in form, and it was almost certainly understood by the commentator as a third-figure *categorical* syllogism. ('Almost certainly', because the formulation is loose—but so are most comparable formulations by the later Platonic commentators, who often put Plato's arguments into Aristotelian categorical figures and who rarely bother to make the quantifying phrases explicit.) Diels-Schubart themselves

## XIII

Alexander discusses each of the three hypothetical figures in turn. He illustrates his first figure by the valid mood:

(1) If A, then B
If B, then C
———
If A, then C.

And he observes that in the case of (1) 'the conclusion can also be taken the other way about . . .—the other way about not without qualification but including negation' (326. 37–327. 1). Taking 'If A, then C' the 'other way about' means converting it to 'If C, then A'; taking it the 'other way about . . . including negation' means contraposing it to 'If not-C, then not-A'.[1] Thus Alexander recognizes, in addition to (1), the argument pattern:

(2) If A, then B
If B, then C
———
If not-C, then not-A.

And he derives the validity of (2) from that of (1) by way of a rule of contraposition. Alexander gives no other illustrations of first figure syllogisms. He considers no invalid moods.

Turning to his second figure, Alexander reports that 'the pairing is syllogistical if the apodosis of the two antecedents is taken in contradictory fashion' (327. 7–8). That must represent the metatheorem that a premiss-pairing of form (P2) yields a conclusion by (H**) if and only if[2] it takes one or other of the forms:

($\alpha$) If A, then C; if B, then not-C
($\beta$) If A, then not-C; if B, then C

That observation is true, and Alexander truly remarks that in such cases 'if one of the antecedents holds, the other does not'.

From ($\alpha$) and ($\beta$), then, we can produce four valid moods. Alexander contents himself with one:

(3) If A, then C
If B, then not-C
———
If A, then not-B.

refer to the *Aristotelian* figures which they seem to have confused with the hypothetical figures in *Didasc.* (Note that the argument is invalid, however it is formalized.)

[1] At 326. 38 I read *ὥcτε μὴ ἑπόμενον εἶναι* ⟨*τὸ Γ*⟩ *ἀλλ' ἡγούμενον*.

[2] At 327. 7 Alexander's *ἄν* is reasonably construed as implying equivalence: 'if' is frequently so used in natural language.

(The schema of the mood is garbled in the manuscripts at 327. 11–12, but the illustration at 327. 12–13 proves that Alexander has mood (3) in mind.)[1]

The account of Alexander's third figure parallels that of his second. He first remarks that premiss-pairings of the form (P3) yield a conclusion if and only if they take one of the forms:

(γ) If A, then B; if not-A, then C
(δ) If not-A, then B; if A, then C

He remarks that 'if one of the consequents does not hold, the other does' (327. 17–18). He points to the two moods thereby generated from pairing (γ). He produces a concrete illustration of schema:

(4) If A, then B
If not-A, then C
———————
If not-B, then C.[2]

## XIV

Alexander does not claim to give an exhaustive account of the wholly hypothetical moods. He is content to characterize each of

[1] The MSS give: *εἰ γὰρ τὸ A, τὸ B· εἰ τὸ Γ, οὐ τὸ B· εἰ ἄρα τὸ A, οὐ τὸ B.* That is nonsense. Wallies prints Prantl's text: *εἰ γὰρ τὸ A, τὸ Γ· κτλ.* But that yields a *first* figure syllogism. (Prantl must suppose that Alexander is citing—without explanation— a first-figure mood to which (3) can be reduced. That seems to me wholly implausible.) I read: *εἰ γὰρ τὸ A, τὸ Γ· εἰ τὸ B, οὐ τὸ Γ· εἰ ἄρα τὸ A, οὐ τὸ B.*

[2] Philoponus' account of the second and third figures is puzzling. The text reads: *γίνεται πάλιν δεύτερον* ⟨sc. *cχῆμα*⟩ *ὅταν οὕτωc εἴπω· εἰ τὸ A, καὶ τὸ B· εἰ μὴ τὸ Γ, οὐδὲ τὸ B· εἰ μὴ τὸ A ἄρα, οὐδὲ τὸ Γ. ὁμοίωc δὲ καὶ τὸ τρίτον οὕτωc· εἰ μὴ τὸ B, οὐδὲ τὸ A· εἰ τὸ B, καὶ τὸ Γ· εἰ μὴ τὸ A ἄρα, οὐδὲ τὶ τῶν Γ. δυνατὸν δὲ καὶ κατηγορικὰc λαβεῖν ἀμφοτέραc* (*in A.Pr.* 302. 20–3). It is plain that Philoponus follows *Alexander's* ordering of the hypothetical figures. But his two illustrations are both invalid. Now an invalid inference will serve just as well as a valid one to illustrate a figure—and Philoponus' illustrations have the appropriate *form.* Yet it is difficult to avoid the suspicion that Philoponus supposes himself to have been producing *valid* illustrations, a suspicion which is confirmed by the note *δυνατὸν δέ κτλ.* For Philoponus means, I take it, that in the third figure both elements in the conclusion may be affirmative. (In fact the syllogism 'If not-A, then not-B; if A, then C: therefore, if B, then C' is valid.) Hence we should paraphrase the note thus: 'You can also ⟨produce a (valid) syllogism in the third figure⟩ by taking . . .'. It may be objected that *λαβεῖν* should refer to the *premisses,* so that Philoponus is alluding to the inference 'If A, then B; if A, then C: therefore, if B, then C'. And that inference is invalid. But even if that uncharitable objection is correct, the *δυνατόν* sentence still suggests that Philoponus imagines he is producing *valid* illustrations. Either the Greek text is badly corrupt or (more probably) Philoponus has made a logical howler.

the figures and to indicate one or two of the valid moods in each. As a matter of fact, in each figure there are four possible schemata for the first premiss and four for the second. (In the first figure they are: (1) If E, then M; If E, then not-M; If not-E, then M; If not-E, then not-M; (2) If M, then E*; If not-M, then E*; If M, then not-E*; If not-M, then not-E*.) Again, there are eight possible forms for the conclusion: If E, then E*; If E, then not-E*; If not-E, then E*; If not-E, then not-E*; If E*, then E; If E*, then not-E*, If not-E*, then E; If not-E*, then not-E. Since there are three figures, there are $3 \times 4 \times 4 \times 8 = 384$ moods. Of those 384 moods, forty-eight are valid.[1]

The fullest ancient treatment of wholly hypothetical syllogistic is found in Boethius. His *de hypotheticis syllogismis*[2] considers the three figures[3] and their constituent moods in a systematic

[1] Dürr, op. cit. (p. 294 n. 3), p. 52, observes that Boethius holds the premiss-pair 'If A, then B; if A, then C' to yield no conclusion. He then accuses Boethius of failing to see the validity of:

If A, then B
If A, then C
―――――――――
If A, then B and C.

It is true that Boethius does not mention that inference. But it is not a wholly hypothetical syllogism as he and his Peripatetic predecessors understood the phrase, i.e. it is not an example of (H**). (See also above, p. 296 n. 1). In a complete logic of conditionals (such as the Stoics perhaps tried to construct) inferences of that type have their place: the Peripatetics, however, make no claim to construct a complete logic of conditionals.

[2] II. ix. 1–III. vi. 4: see Dürr, op. cit. (p. 294 n. 3), pp. 43–55; on Boethius' logic see also H. Chadwick, *Boethius* (Oxford, 1981), ch. 3; J. Barnes, 'Boethius and the Study of Logic', in M. Gibson (ed.), *Boethius* (Oxford, 1981).

[3] Boethius' treatment of the first figure is odd, for his schemata are not, strictly speaking, instances of the form (H**). Thus in place of (1) he offers:

(1*) (If A, then B) and (if B, then C)
A
―――――――――
C

In general, he *conjoins* the two hypothetical premisses of the Theophrastan syllogisms by the word *et* (but occasionally he omits to do so), and he supplies as an additional *premiss* the antecedent of Theophrastus' hypothetical conclusion. His *conclusions* are then the consequents of the Theophrastan conclusions. He observes that (1*) and its congeners are 'imperfect' syllogisms, and says that 'their proof is a demonstration through a syllogism': the syllogisms in question are in fact (1) and its congeners (*hyp. syll.* II. ix. 4–5; cf. 2). Thus Boethius' first figure consists in fact of *mixed* hypothetical syllogisms, which he grounds on the *wholly* hypothetical syllogisms of the first figure. (Abelard, who follows Boethius closely, notices this point: see Petrus Abaelardus, *Dialectica*, ed. L. M. de Rijk (Assen, 1956), p. 517. But Abelard treats *all* the figures on the model of fashion: he describes the possible premiss-pairings and the possible conclusions; he determines the valid moods; he attempts to show, by formal proof or by counter-example, the invalidity of the invalid moods. In all he gives explicit and detailed attention to 192 of the 384 moods, and he comments summarily on a further 128.[1]

Boethius' treatise is largely Peripatetic in content and in character. It is true that he claims originality for his work on the subject: hypothetical syllogistic, he says, is a study 'which I have found discussed briefly and confusedly by a very few Greek authors—and by no Latin authors at all. . . . Aristotle wrote nothing on the matter. Theophrastus, a man of exhaustive learning, dealt only with the elements of the subject. Eudemus followed a broader path of study—yet even he seems as it were to have sown the seedbeds without seeing the fruit' (*hyp. syll.* I. i. 3). But it is hard to believe Boethius' story, and it is likely that in *hyp. syll.*, as in his other logical writings, he is relying heavily on Greek sources.[2] How far he introduced minor novelties we cannot yet

Boethius' first figure.) In the second and third figures Boethius presents standard wholly hypothetical moods. (There are occasional traces of the first-figure style of treatment: III. ii. 6, iii. 2, iv. 6, vi. 2.) Boethius gives no indication that he is aware of this lack of uniformity in his treatment: he purports to be giving a uniform account of wholly hypothetical syllogistic. The oddity is best explained by the hypothesis that Boethius made use of more than one immediate source (see below, n. 2). Three further oddities in his treatment of the first figure point in the same direction. (i) The reference to the three figures at II. ix. 3 breaks the train of thought. (ii) The brief and unexplained allusions to 'extra' syllogisms at II. x. 7 and xi. 1 suggest an idiosyncratic source for the account of the first figure. (iii) sixty-four of the 128 moods of the first figure are wholly ignored (see below, n. 1).

[1] In the second and third figures Boethius lists the eight 'equimodal' premiss-pairings (i.e. the premiss-pairings in which the middle term is twice +M or twice −M), and he observes that it is easy to show that none of them yield valid syllogisms (III. iii. 7–iv. 1, ix. 2–4); but he does not discuss the equimodal pairings individually. In the first figure he does not even mention the eight premiss-pairings which yield no valid syllogisms, and he thus ignores completely sixty-four moods.

[2] For Boethius' sources see Dürr, op. cit. (p. 294 n. 3), pp. 4–15; Obertello, op. cit. (p. 286 n. 2), pp. 15–66: G. Striker, 'Zur Frage nach den Quellen von Boethius' *de hypotheticis syllogismis*', *AGPh* 55 (1973), 70–5; M. Maróth, 'Die hypothetischen Syllogismen', *Acta Antiqua*, 27 (1979), 407–36. Some scholars may find it improper to accuse Boethius, as in effect I do, of a gross exaggeration, if not a downright lie, in his claim to originality. And they may observe that *Quellenforschung*, never a simple task, is peculiarly difficult in the case of formal logic, where, as the history of the subject shows, independent discoveries of the same facts are perfectly possible. Even if we had a text of Theophrastus which coincided in content with that of Boethius, we could not safely infer that

say.[1] For my part, I am inclined to suppose that most of Boethius' account was already in Theophrastus, and that Alexander was excerpting from a passage which Boethius reproduced, indirectly, in fuller form.[2]

However that may be, in Boethius' *hyp. syll.* we possess a comprehensive account of wholly hypothetical syllogistic as the Peripatetics saw it.[3]

## XV

Having discussed the figures, Alexander turns to their 'generation'. Aristotle's second and third categorical figures are 'generated' from the first by 'conversion': similarly, Alexander's second and third hypothetical figures are 'generated' from the first by 'conversion'. The point is trivial. The first categorical figure is determined by the pairing:

AxB, BxC[4]

'Conversion', in the present context, is the interchange of subject and predicate terms.[5] Thus by 'converting' AxB we generate

BxA, BxC

Boethius had copied—at first or second hand—the earlier treatment: he might easily have hit upon it independently. The detailed study of Boethius' style and method which is necessary for any serious *Quellenforschung* has not yet been carried out. But the preliminary work (by Striker and by Maróth) strongly suggests that Boethius was basing himself on more than one Greek source. Perhaps I should add that there is nothing discreditable in that; nor does it imply that Boethius was a mere copyist. (See further Barnes, op. cit. (p. 301 n. 2).)

[1] Here again the Arabic commentators, whose works have as yet scarcely been studied, will probably offer further intelligence: see Maróth, op. cit (p. 302 n. 2).

[2] But I do not suppose that Boethius had a copy of Theophrastus: he reproduced Theophrastus at second or third hand, and by way of at least two distinct intermediary sources (see above, p. 301 n. 3).

[3] Boethius shows no interest in 'reducing' hypothetical to categorical syllogistic; nor does he advert to the 'analogies' between the two types of syllogistic. (But Abelard does: op. cit., p. 301 n. 3, pp. 517–18, 522.)

[4] Here and hereafter I follow the convention established by Patzig, op. cit., p. 296 n. 2, whereby the predicate term precedes the subject term. Thus AaB represents 'A belongs to every B' or 'Every B is A'.

[5] At *in A.Pr.* 29. 23–7 Alexander distinguishes it from other sorts of 'conversion' as *ἀντιστροφή τῶν ὅρων*. Galen sensibly uses a separate term, *ἀναστροφή*, for this form of 'conversion', and he warns against the danger of confusing *ἀναστροφή* with *ἀντιστροφή* proper, or contraposition: *inst. log.* vi. 3–4, *simp. med. temp.* xi. 500K.

which is the pairing characteristic of the second categorical figure. By 'converting' BxC we generate

AxB, CxB

which is the third figure. In hypothetical syllogistic, 'conversion' is the interchange of consequent and antecedent. By 'converting' the second premiss of the first figure pairing

If $\pm$E, then $\pm$M; if $\pm$M, then $\pm$E*

we 'generate':

If $\pm$E, then $\pm$M; if $\pm$E*, then $\pm$M

which is Alexander's second figure. Similarly, by 'converting' 'If $\pm$E, then $\pm$M' we 'generate':

If $\pm$M, then $\pm$E; if $\pm$M, then $\pm$E*

which is Alexander's third figure.

## XVI

'Similarly, arguments in the second and third figures will be analysed into the first figure' (327. 33–4). *This* point, which Alexander does not elaborate, is not trivial; for analysis is a matter of reduction or derivation,[1] and Alexander is claiming that the valid moods of the latter two figures can be derived from the valid moods in the first figure.

The derivations are simple enough, requiring no more logical apparatus than a rule of contraposition (which we have already seen invoked within the first figure). Take syllogism (3), from the second figure. Its second premiss, 'If B, then not-C', gives by contraposition 'If C, then not-B'; and that together with the first premiss, 'If A, then C', forms a first-figure pairing which yields the conclusion 'If A, then not-B'.[2] That derivation is given by Boethius (*hyp. syll.* III. iv. 3). His text contains many similar derivations.[3]

[1] For this sense of ἀναλύειν, in which it is a synonym of ἀνάγειν, see *in A.Pr.* 7. 25; cf., for example, *A.Pr.* 50b33. (See above, p. 286 n. 3.)

[2] More formally:

| | | |
|---|---|---|
| (i) | If A, then C | Assumption |
| (ii) | If B, then not-C | Assumption |
| (iii) | If C, then not-B | (ii), Contraposition |
| (iv) | If A, then not-B | (i), (ii), first fig. syllog. |

[3] All of Boethius' derivations rest upon the single application of a rule of contraposition which is, in effect, the following principle:

From 'If $\pm$A, then $\pm$B', infer 'If $\pm$B*, then $\pm$A*'

Aristotle derives the valid moods of the second and third categorical figures from the valid moods of the first. Compared to Aristotle's derivations, the derivations required by hypothetical syllogistic are child's play. Theophrastus' account of wholly hypothetical syllogisms was clearly patterned on Aristotle's account of categorical syllogistic. Theophrastus must surely have wondered if hypothetical logic allowed for the same sort of reductions as categorical logic. He can scarcely have avoided the correct answer: I suppose that the derivations we find in Boethius were discovered by Theophrastus.[1]

## XVII

'These, then, are the simple and primary so-called wholly hypothetical arguments. All the compound wholly hypothetical arguments will be proved to be constituted from them' (328. 1–2). Boethius offers as a type of compound hypothetical proposition the schema:

If if A then B, then if C then D

(see *hyp. syll.* I. v. 1). Compound hypothetical syllogisms may then have been those with compound hypothetical premisses—premisses in which the replacements for 'A' and 'B' in 'If A, then B', are themselves hypothetical propositions. It is of course true that any compound syllogisms of that sort are 'constituted from'—or are substitution instances of—the simple syllogisms we have been discussing.

The *Didascalicus* (p. 159 H.), however, suggests another interpretation of 'compound'. The passage offers three Platonic illustrations of hypothetical syllogisms, one from each figure. The text for the third figure is probably corrupt.[2] The

—where $\pm X^*$ is not-X if $\pm X$ is X and X if $\pm X$ is not-X. Usually his application of the principle is tacit; but sometimes he explicitly refers to it: III. i. 4, 6, ii. 1, 2, 5, iv. 5, 6, 7, 8, v. 1, 2, 3, 4, 5, 6, 7, vi. 1. He never actually *says* that he is reducing second- and third-figure syllogisms to the first figure, but that is evidently what he is in fact doing (so, explicitly, Abelard, op. cit. (p. 301 n. 3), p. 522).

[1] What of reductions *within* the figures? Boethius does not consider them. Alexander, as we have seen, reduces (2) to (1) in the first figure. In fact *all* the valid hypothetical syllogisms can be reduced to syllogism (1). All that is required for a reduction is (i) the extended rule of contraposition (above, p. 304 n. 3) and (ii) a rule of substitution: If $\Sigma$(A) is a valid syllogism containing the component A, and $\Sigma$(not-A) results from the uniform replacement in $\Sigma$(A) of A by not-A, then $\Sigma$(not-A) is valid.

[2] The published text cites two premisses (of the form 'If not-A, then B; if A,

first two illustrations present arguments of the following forms:

(5) If not-A, then either not-B or not-C or not-D
If either not-B or not-C or not-D, then not-E
If not-E, then not-F
___
If not-A, then not-F.

(6) If not-A, then not-B and not-C
If D, then either B or C
___
If not-A, then not-D.

Argument (6) evidently supposes that 'not-B and not-C' is negated by 'either B or C'. Given that supposition, the argument is a substitution instance of a third-figure hypothetical. Argument (5) is more interesting. It is a substitution instance of

(7) If A, then B
If B, then C
If C, then D
___
If A, then D.

And (7) is a 'compound' hypothetical syllogism in a new sense: unlike the simple hypotheticals, it has more than two premisses.

Plainly, (7) can be 'constituted' from two simple first-figure hypotheticals. From the first two premisses of (7) we can infer, by argument (1), that if A, then C. From that conclusion and the third premiss of (7), argument (1) again yields the conclusion that if A, then D. It is possible that Alexander's allusion to 'compound' arguments is, or encompasses, a reference to syllogisms like (7), and that he is claiming to be able to prove that all hypothetical syllogisms with a plurality of premisses can be reduced to sequences of two-premissed hypotheticals. We cannot be sure if that is his meaning, or if Theophrastus discussed such arguments. If the *Didascalicus* is by Albinus, then it seems probable that compound arguments like (7) had been noticed by Peripatetic logicians before Alexander.[1]

then C') and omits the conclusion: no doubt 'Alcinous' originally wrote out the conclusion too.

[1] Alexander does not mention the possibility of *modal* hypothetical syllogisms. Boethius does (*hyp. syll.* I. viii. 6–7), but he decides not to investigate them on the grounds that they are rarely used (I. ix. 3).

## XVIII

A generous, but not wholly implausible, reconstruction of Theophrastus' account of hypothetical syllogisms would thus ascribe the following discoveries to him. He investigated inferences of the form (H**). He arranged them into three figures. Within each figure he discussed, systematically, the possible moods, singling out those which were valid. He asserted, if he did not prove, certain metatheorems about hypothetical syllogistic. He showed that the second- and third-figure moods could be derived from the first figure. He produced some derivations within the figures. He may perhaps have said something about compound syllogisms.

Modern logicians will not perhaps regard that achievement as particularly remarkable. Yet in its day it represented a definite extension of logical science beyond anything that is found in Aristotle's *Organon*.[1] At the same time, it is clear that Theophrastus was relying upon his master's work: Aristotle's treatment of categorical syllogistic gave him a model and a pattern for his own treatment of wholly hypothetical syllogisms.

## XIX

Alexander discusses Theophrastus' account of wholly hypothetical syllogisms in the context of the Peripatetic programme: 'wholly hypothetical syllogisms will be thought not to be amenable to proof by selection' (326. 8–10)—are these arguments amenable to any other form of reduction?

Alexander first offers a suggestion of his own. 'Perhaps these arguments are not syllogisms in the proper sense' (326. 12). The paragraph which that cautious sentence introduces has been widely misunderstood. Scholars have connected it with Alexander's preceding explanation of Theophrastus' phrase 'in virtue of an analogy'. They have supposed Alexander to mean: 'Perhaps Theophrastus called these arguments "arguments in virtue of an analogy" because they are not really syllogisms at all . . .'.[2] That is mistaken. Alexander is introducing his own solution to the

[1] Wholly hypothetical syllogistic does not comprise the sum of Theophrastus' achievement in non-categorical logic: see Barnes, op. cit. (p. 285 n. 4), for further details (and for a brief argument against the tempting conclusion that Theophrastus the Peripatetic invented Stoic logic).

[2] So, for example, H. Maier, *Die Syllogistik des Aristoteles*, i (Tübingen, 1890), 280; Bocheński, *La Logique* . . . (p. 290 n. 1), p. 116; W. C. and M. Kneale, *The Development of Logic* (Oxford, 1962), pp. 110–11; Graeser, op. cit. (p. 291 n. 1), p. 98; Repici, op. cit. (p. 291 n. 1), p. 148.

question he posed at 326. 8, a solution that will shortly be followed by Theophrastus' answer to the same question.[1]

Alexander's answer is this: 'Wholly hypothetical syllogisms are not genuine *syllogisms*. For they do not *prove* anything. They do not prove anything because their conditional conclusions do not *assert* anything—they do not say that anything holds or does not hold. Hence the fact that they do not reduce to the categorical figures does not show that some *syllogisms* do not reduce to the categorical figures.' Alexander says the same thing at 265. 15–17. He thought that he was expressing Aristotle's own view on the matter (see 390. 18–19).[2]

That solution may seem to be a mere quibble. For surely the conclusion of a wholly hypothetical syllogism *does* assert something, namely a conditional proposition of the form 'If P, then Q'. But in fact the standing of Alexander's solution depends upon the nature of conditional sentences—or rather, upon the nature of those conditional sentences which feature in the conclusions of wholly hypothetical syllogisms.[3] There are modern philosophers who have shared Alexander's view that conditional sentences do not, in their primary use, express genuine propositions.[4] Just as a conditional bet or a conditional promise is not (yet) a bet or a promise, so a conditional assertion is not (yet) an assertion. If the condition is fulfilled, the bet is on, the promise undertaken, the assertion made; but unless and until the condition is fulfilled, there is no bet, no promise, no assertion. Thus the conditional 'conclusion' of a hypothetical argument is not (yet) a genuine conclusion, and so the hypothetical argument is not (yet) a genuine syllogism.

Whatever the merits of his own solution, Alexander appears to subscribe as well to the solution offered by Theophrastus. For 'Theophrastus has proved in the first book of his *Prior*

[1] Philoponus understood Alexander correctly (*in A.Pr.* 302. 12–15); but his own answer to the problem of 'reduction' (302. 23–32) is neither Alexander's nor Theophrastus'.

[2] Cf., therefore, Striker, op. cit. (p. 285 n. 1), Lear, op. cit. (p. 285 n. 1).

[3] Alexander's solution also assumes that the conclusion of any syllogism must be a genuine assertion: were his implicit account of the nature of conditionals accepted, that assumption might well be questioned.

[4] See, for example, J. L. Mackie, *Truth, Probability and Paradox* (Oxford, 1973), p. 93: 'I want to offer, then, this general analysis: *to say 'If P, Q' is to assert Q within the scope of the supposition that P*. . . . ⟨This analysis⟩ abandons the claim that conditionals are in a strict sense statements, that they are in general any sort of descriptions that must either be fulfilled or be not fulfilled by the way things are, and hence that they are in general simply true or simply false.'

*Analytics*'[1] that wholly hypothetical syllogisms 'reduce, in another way, to the three figures' of categorical syllogistic (326. 21).[2] By 'another way', Alexander means a way other than the 'method of selection'.

## XX

At the heart of the Theophrastan method lies an analogy. Alexander repeatedly stresses that there are close analogies between hypothetical and categorical syllogistic: in each there are three figures, determined by the position of the 'middle term'; in each the second and third figures are 'generated' from the first; in each the second and third figures 'analyse' into the first. 'As it is in categorical syllogisms, so it is in wholly hypothetical syllogisms.'

The basis of that general analogy is a particular analogy to which Alexander draws attention at the beginning of his discussion. 'Being a consequent or apodosis is analogous to being predicated, and being antecedent to being subject—for in a way it is subject for what is inferred from it' (326. 31–2).[3] Consider the (indefinite) categorical proposition 'Man is an animal' or 'Animal is predicated of man', and the hypothetical proposition 'If he is a man, he is an animal'. There is an analogy, Alexander maintains, between the relation in which *man* stands to *animal* in the categorical proposition and the relation in which *he is a man* stands to *he is an animal* in the hypothetical proposition. (And, trivially, there is an analogy between the converses of the two relations.)

No doubt there is an analogy. It is presumably no accident that Aristotle used the word 'follow' as a synonym of 'be predicated of' in his categorical syllogistic,[4] or that the metalogical vocabularies of hypothetical and categorical syllogistic overlap. Modern sentence logicians are equally aware of the analogy. In his paper on 'Logical Generality', Frege observes that:

> We have various expressions for the same general thought:
>
> All men are mortal
> Every man is mortal
> If something is a man, it is mortal

[1] Cf. Philoponus, *in A.Pr.* 302. 5.
[2] Cf. Philoponus, *in A.Pr.* 302. 15; [Ammonius], *in A.Pr.* 67. 24–30.
[3] Cf. Philoponus, *in A.Pr.* 302. 17–19.
[4] So ἕπεσθαι at, for example, *A.Pr.* $43^{b}3$, $44^{a}13$, $56^{a}20$; ἀκολουθεῖν at, for example, $26^{a}2$, $43^{b}4$ (see Alexander, *in A.Pr.* 55. 10).

> The differences in the expression do not affect the thought itself. . . . In the last mode of expression we have the form of the hypothetical compound sentence—a form we can hardly avoid using in other cases too. . . .[1]

Frege's analysis of generality—which is the basis of his development of predicate logic—turns on his perception of an analogy between categorical and hypothetical sentences. Frege in effect appeals to the analogy to reduce categorical propositions to hypotheticals. Alexander had appealed to the same analogy with the opposite intention.

## XXI

What is the weight of Alexander's analogy? Precisely how is it supposed to ground a reduction of wholly hypothetical syllogisms to the three categorical figures? Alexander remarks that 'the combinations in ⟨wholly hypothetical⟩ arguments, being similar in this way to those in the categorical figures, should reasonably reduce to them' (327. 20–1). He says no more: he does not actually produce a reduction or even indicate how a reduction might be carried out; he merely says that it is reasonable, given the analogy, to suppose that a reduction is available.[2]

Then did Theophrastus merely develop certain analogies and infer that a reduction was in principle possible? I think not.[3] First, Alexander expressly says that 'Theophrastus *has proved*' that hypothetical syllogisms 'reduce *in another way*' to categorical syllogistic (326. 21). That would be a remarkably inaccurate way of reporting the fact that Theophrastus had opined that some reduction was in principle possible. Secondly, there is reason to

[1] G. Frege, 'Logical Generality', in his *Posthumous Writings* (Oxford, 1979), p. 259.

[2] Sommers' investigations lead him to uphold 'the parity and mutual independence of term and propositional logic.' He maintains that categorical and hypothetical propositions 'must share a *common* structure'; he discovers 'important formal affinities' between the two; he talks of 'isomorphism' ((op. cit., p. 281 n. 2), p. 160; see below, p. 317 n. 3). The 'affinities' of Sommers are the 'analogies' of Alexander. But whereas Alexander takes his analogies as evidence that hypothetical syllogisms can be reduced to categoricals, Sommers eventually concludes that his affinities and isomorphisms indicate a common abstract structure of which hypothetical and categorical syllogisms are distinct specifications.

[3] Thus when Alexander says that hypothetical syllogisms 'should reasonably reduce' to the categorical figures, I take him to mean something like this: 'In the light of the pervasive analogies I have drawn attention to, it is only reasonable that there should be a reduction of the sort which Theophrastus proved.'

think that the careful development of the analogy was the work of Alexander rather than of Theophrastus.

Alexander's ordering of the hypothetical figures is, as we have seen, different from Theophrastus'. And Alexander's ordering was plainly chosen with an eye to the analogy with categorical syllogistic. The second categorical figure is determined by the pairing

BxA, BxC

in which the middle term, B, appears twice in predicate position. Given the analogy between predicate and consequent, the second hypothetical figure should be determined by the pairing

If $\pm$E, then $\pm$M; if $\pm$E*, then $\pm$M.

And that is indeed Alexander's second figure.

That point is entirely obvious. Had Theophrastus been concerned to stress the analogy between hypothetical and categorical syllogistic he could hardly have failed to adopt Alexander's ordering of the hypothetical figures.[1] The fact that he did not do so makes it unlikely that he was concerned to elaborate the Alexandrian analogies.

It might be asked why Theophrastus originally chose the order he did. Some scholars have suggested that syllogisms in Theophrastus' third (Alexander's second) figure require more operations to reduce them to first-figure syllogisms than do syllogisms in Theophrastus' second (Alexander's third) figure. Theophrastus' second figure is second because it is, so to speak, logically closer to the first figure.[2] That is not a compelling argument, as a glance at Boethius' derivations shows. All the reductions require *one* operation—an application of the extended principle of contraposition[3]—and nothing more: second and third figures are on an equal footing, equally close to the first figure.

I suspect that Theophrastus had *no* reason for choosing the order he did. The order of the second and third figures seemed logically indifferent to him, and no cunning thought lies behind his choice of arrangement.

## XXII

I said that Frege perceived an analogy between categorical and hypothetical sentences. That is not quite accurate: Frege asserted

[1] Obertello, op. cit. (p. 286 n. 2), p. 143, wrongly says that *Theophrastus* follows the order of the Aristotelian figures.

[2] So Bocheński, *La Logique* . . . (p. 290 n. 1), p. 115.

[3] See above, p. 304 n. 3.

that certain categorical sentences express the very same thought as certain hypotheticals. He saw an identity, not merely an analogy, here.

Frege's view is an ancient one. It is found, for example, in the writings of Galen, who took a peculiar and justified pride in his knowledge of logic. Thus when he is criticizing an argument which draws on the two premisses 'All pungent things produce hoarseness' and 'Oil produces hoarseness', Galen comments that 'from these it does not follow that oil is pungent, *whether we make the premisses categorical or hypothetical*' (*simp. med. temp.* xi. 499K). He thus supposes that one and the same proposition can be 'made' either categorical or hypothetical. In other words, we may represent the thought expressed, informally, by the sentence 'Oil produces hoarseness' *either* by the categorical:

> Hoarseness-production belongs to all oil

or by the hypothetical:

> If it is oil, then it produces hoarseness.

Consider, then, Alexander's illustrative syllogism in the hypothetical mood (1). Its first premiss,

> If he is a man, then he is an animal

could be taken as a formal version of the informal 'Man is an animal'. And the same thought can be expressed equally well by the categorical:

> Man belongs to every animal.

But then the first hypothetical mood 'reduces' easily to categorical syllogistic. For the concrete illustration of (1) can be formalized indifferently as:

(1H) If he is a man, he is an animal
If he is an animal, he is a substance
―――――――――――――――――
If he is a man, he is a substance

or as:

(1C) Animal belongs to every man
Substance belongs to every animal
―――――――――――――――――
Substance belongs to every man.

And (1C) is a categorical syllogism in *Barbara* with inverted premisses.

If this account can be generalized, then hypothetical syllogistic will 'reduce' to categorical syllogistic in a strong sense: wholly hypothetical syllogisms will turn out on examination to be notational variants on categorical syllogisms.[1]

Whatever the merits that reduction may possess, it was surely not the reduction that Theophrastus had in mind.[2] For neither Galen nor Frege supposes that conditional sentences *in general* express thoughts which are equally well expressed in categorical sentences. Rather, they both have in mind a particular form of conditional sentence, namely 'If anything is F, it is G'.[3] Thus the reduction will work, if at all, only for hypothetical syllogisms whose component propositions all have that particular form. In other words, it will only work if we construe Theophrastus' wholly hypothetical syllogisms on what I earlier called the Boethian line. But that construe is probably wrong. Hence the reduction suggested by the Galen-Frege thesis is unlikely to have been Theophrastus' reduction.[4]

## XXIII

Let us consider the desired reduction in more abstract terms. If hypothetical propositions are to be reduced to categorical form, then two—or perhaps three—requirements must be satisfied. First, the sentential operator 'If. . ., then ——' must somehow be translated into an operator on terms. Secondly, the sentential components of hypothetical propositions must somehow be re-construed as terms.[5] And perhaps, thirdly, the sentential operator

[1] Compare Galen's remarks, *inst. log.* xix. 1–3, on syllogisms κατὰ πρόςληψιν.

[2] At 326. 32 Alexander says that the antecedent of a conditional 'in a way . . . is subject for what is inferred from it'. Without the qualifying phrase 'in a way (πωc)' that sentence would constitute an endorsement of the Galen-Frege identity thesis; but we should not suppress the qualification (see below, p. 315 n. 3).

[3] Strictly speaking, propositions of the form

If anything is F, it is G

are not conditionals. They do not exhibit the general form 'If A, then B'; they are not constructed by conjoining two propositions by way of the operator 'If . . ., then ——'. Galen is nevertheless prepared to call such sentences 'conditionals' or cυνημμένα (and so, I think, were most ancient logicians). Frege is more careful. When he says that 'in the last mode of expression we have the form of the hypothetical compound sentence' (see above, p. 310), he does not mean that the last mode of expression *is* a hypothetical compound sentence: he means that it *contains* an embedded conditional, namely the open sentence 'If x is a man, then x is mortal'.

[4] There is a further argument to that conclusion, derivable from the text of Boethius himself: see below, p. 316.

[5] See Leibniz, *Generales Inquisitiones de analysi notionum et veritatum*, § 75 [in L. Couturat, *Opuscules et Fragments inédits de Leibniz* (Paris, 1903), p. 377]: *si, ut*

of negation must be displayed as a negation operator within term logic.[1]

The third requirement can be met in more ways than one. Categorical syllogistic has two modes of introducing negation into its propositions: you may use one of the two negative term-relations, *e* and *o*; and you may negate the terms themselves.[2] Those two categorical modes of negativity will surely suffice for the reduction of sentential negation.

The second requirement, similarly, can be satisfied by various devices.[3] We might, for example, construct from any sentence 'P' the one-place predicate '. . . is such that P'. Like any one-place predicate, '. . . is such that P' will generate a term. For example, from the sentence 'Verdi is Italian' we construct the

*spero, possim concipere omnes propositiones instar terminorum, et hypotheticas instar categoricarum, et universaliter tractare omnes, miram ea res in mea characteristica et analysi notionum promittit facilitatem, eritque inventum maximi momenti.*

[1] Wholly hypothetical syllogistic does not consider negated conditionals of the form 'It is not the case that (if A, then B)'. Negation only appears as an operator on the components of the hypothetical propositions. Thus we might decide to ignore the internal structure of the components of, say, 'If not-A, then not-B': we might be satisfied if we could reduce 'not-A' and 'not-B' to terms, and relinquish any hope of reflecting in categorical form the structural difference between 'If not-A, then not-B' and 'If A, then B'. But in that case the principle of contraposition (above, p. 304 n. 3) will have a peculiar status: it will have no formal perspicuity and will appear a puzzling oddity. For it will have the general form: 'From "If A, then B" infer "If C, then D", when A, B, C, D, are of such and such a type.'

[2] Negative terms are not treated in the formal development of Aristotle's syllogistic in *A.Pr. A* 2–7. But they are extensively discussed in *A.Pr. A* 46, and elsewhere Aristotle propounds some rules concerning them. (See especially *int.* $20^{a}20$–3: if $\bar{A}aB$, then AeB; if AiB, then $\bar{A}oB$; *Top.* $113^{b}15$–26: if AaB, then $\bar{B}a\bar{A}$. See further Thom, op. cit. (p. 296 n. 1), pp. 125–8.) Theophrastus is known to have evinced some interest in the matter: he coined the phrase *πρότασις κατὰ μετάθεσιν* (or *ἐκ μεταθέσεως*) for propositions which predicate negative terms, a phrase which was generally adopted by the later Peripatetics (see Alexander, *in A.Pr.* 397. 2–4: other texts collected under Theophrastus, F 8 G). A theory of negative terms was extensively developed in later antiquity (see, for example, A. N. Prior, 'The logic of negative terms in Boethius', *Franciscan Studies*, 13 (1953), 1–6): it is quite possible that the development originated with Theophrastus.

[3] What is needed is a function which makes terms from propositions in such a way that the semantic content of the term is the same as that of the proposition. Thus Sommers, who thinks that to every proposition there corresponds a (possible) state of affairs, introduces the term-forming function '[ ]': where 'p' is a proposition, '[p]' is the corresponding term—and '[p]' should be read as 'state of affairs in which p' ((op. cit., p. 281 n. 2), p. 153; cf. Castañeda, op. cit. (p. 281 n. 1), p. 491).

predicate '. . . is such that Verdi is Italian', which generates the term *being such that Verdi is Italian.* I shall use bold type to mark such propositional terms: in general, **P** is the term (*being such that P*), produced from the sentence 'P'. Plainly, **P** is true of any object just in case that object is such that P; and, trivially, an object is such that P just in case P. For example, **Verdi is Italian** is true of Berlioz just in case Berlioz is such that Verdi is Italian, and Berlioz is such that Verdi is Italian just in case Verdi is Italian.[1]

The first requirement demands the representation of the sentential connective 'If . . ., then ——' by means of some term relation. More precisely, 'If . . ., then ——' must in some fashion be reduced to the relation of term predication together with a sign of quantity. Within Aristotle's syllogistic, that amounts to the requirement that the conditional connective be transmuted into one of the four relations *a*, *e*, *i*, and *o*.[2] A moment's thought shows that the only plausible candidate is *a*, the universal affirmative relation. Thus 'If A, then B' becomes 'Everything such that A is a thing such that B' or **BaA.**[3]

Consider, then, the first hypothetical mood, (1). That is 'reduced' to:

$$(1^{*})\quad \begin{array}{l}\mathbf{BaA}\\ \underline{\mathbf{CaB}}\\ \mathbf{CaA}\end{array}$$

[1] The term **Verdi is Italian** must seem highly artificial—a logician's contrivance, analogous to the modern contrivance of treating a complete sentence as a zero-placed predicate. The Peripatetics recognized the need to construct artificial terms, and they apparently felt no embarrassment at their contrived nature: see, for example, Aristotle's discussion at *A.Pr. A* 36–8, and, for a good example, Alexander, *in A.Pr.* 344. 9–355. 12 (cf. Thom, op. cit. (p. 296 n. 1), pp. 75–6). For Leibniz see op. cit. (p. 313 n. 5), § 138 [Couturat, p. 389]: *nempe si propositio A est B consideretur ut terminus, . . . oritur abstractum, nempe τὸ A esse B, et si ex propositione A est B sequatur propositio C est D, tunc inde fit nova propositio talis: τὸ A esse B est vel continet τὸ C esse D, seu Beitas ipsius A continet Ceitatem ipsius D, seu Beitas ipsius A est Ceitas ipsius D.*

[2] I ignore two further possibilities, neither of which seems to me to repay close attention. (i) The translation of the conditional might involve a complex set of categorical propositions and a web of term relations. (ii) Categorical syllogistic might be enriched by the addition of further term-connectives—for example, by the connective *m*, where BmA is to be read as 'B belongs to most A'. And some new connective might serve to express 'If . . ., then ——'.

[3] Thus, as Alexander says, the antecedent '*in a way* . . . is subject for what is inferred from it' (see above, p. 313 n. 2). **A,** the term associated with the antecedent A, is subject for **B,** the term associated with the consequent B. So 'in a way' A is subject for B.

which is a categorical syllogism in *Barbara* (with inverted premisses).[1] Again, Alexander's second illustration of first-figure hypothetical syllogisms becomes:

$$(2^*)\quad \begin{array}{l} \mathbf{BaA} \\ \underline{\mathbf{CaB}} \\ \bar{\mathbf{A}}\mathbf{a}\bar{\mathbf{C}} \end{array}$$

That schema is not to be found in Aristotle's *Analytics*, since the *Analytics* does not discuss syllogisms with negative terms. But it is derivable from (1*), given that AaB entails $\bar{\mathrm{B}}\mathrm{a}\bar{\mathrm{A}}$—an entailment which Aristotle states in the *Topics*.[2] And so, it may seem, all the wholly hypothetical moods can be reduced to the categorical figures.

## XXIV

But there is a devastating objection to that reduction. It is implicit in a passage from Boethius' discussion of hypothetical propositions. Boethius first observes that 'if someone asserts "Man is an animal" and then again expresses it thus, "If he is a man, he is an animal", these propositions are admittedly different in style but they do not seem to signify anything different' (*hyp. syll.* I. i. 6). But a little later he indicates that this seeming identity is misleading. 'In a categorical proposition we shall consider the fact that man himself is an animal, i.e. takes on the name of animal; in a hypothetical, we understand that should there be anything that is called a man, it is necessary for there to be something entitled animal. Thus the categorical proposition indicates that the thing it puts as subject takes on the name of the thing predicated; but the hypothetical proposition has this sense—something is the case if something else is (even if neither receives the name of the other)' (*hyp. syll.* I. ii. 2).

Boethius' expression is muddled, but his intention is plain. The categorical proposition 'AaB' implies that B 'takes on the name' of A; that is to say, it implies that there *are* B's which are A. But the hypothetical proposition, 'If it is B, it is A', does not have that implication: it may be true even if there are no B's at all.

Boethius is adverting to a notorious feature of Aristotle's syllogistic. Within Aristotle's system AaB entails BiA (*A.Pr.* $25^{a}17$–19), and BiA entails AiB ($25^{a}20$–2); hence AaB entails AiB. (That last entailment is one of the so-called Laws of Sub-

[1] Cf. Sommers op. cit. (p. 281 n. 2), p. 154.

[2] See above, p. 314 n. 2.

alternation.)[1] Thus **BaA** entails **BiA,** and **BiA** says that some things such that A are such that B. But that is true only if there are some things such that A and some things such that B.[2] **BaA** thus entails that A and that B. But evidently, 'If A, then B' entails neither that A nor that B. Hence **BaA** is not equivalent to 'If A, then B'. The reduction fails.[3]

## XXV

If the old Peripatetic programme is unsuccessful, the underlying cause of its failure is to be found in the fact that AaB in Aristotle's system entails AiB. It will perhaps cause little surprise that subalternation should prove the stumbling-block; for that feature of Aristotle's system has upset logicians of many persuasions and for many reasons.

But categorical syllogistic is not irredeemably Aristotelian in this respect. Logicians have invented systems which, like Aristotle's, are systems of *term* logic, but which, unlike Aristotle's, do not countenance subalternation.[4] One such system was devised by Franz Brentano.[5] In Brentano's syllogistic, the particular

[1] The Laws of Subalternation can be stated as follows:

(a) If AaB, then AiB
(b) If AeB, then AoB

Aristotle states both (a) and (b) in the *Topics* ($109^a3$–6; cf. $119^a34$–6). He states (b), but not (a), in the *Analytics* (*A.Pr.* $26^b15$–16). His syllogistic system commits him to both laws.

[2] **BiA** surely represents 'A and B' (see Sommers, op. cit. (p. 281 n. 2), p. 153). 'A and B' is equivalent to 'Not-(not-A or not-B)'. 'A or B' is equivalent to 'If not-A, then B' (see Boethius, *hyp. syll.* III. x. 4). Thus: A and B = Not-(if not-not-A, then not-B) = Not-($\bar{\mathrm{B}}$a$\bar{\mathrm{A}}$) = Not-($\bar{\mathrm{B}}$aA) = $\bar{\mathrm{B}}$oA = BiA.

[3] Sommers, though generally friendly to the Peripatetic programme, eventually concludes that 'the policy of analysing "if p then q" or "p and q" as a categorical subject-predicate proposition, even if it is a possible one, is not desirable.... Our own standpoint is that "p and q" and "some A is B" ... share a common structure. ... They are analytically autonomous and structurally isomorphous' (op. cit., p. 281 n. 2), p. 159).

[4] In Sommers's TFL system, the Laws of Subalternation do not hold in any straightforward way. Sommers says that 'in TFL "every S is P" is defined as "no S is non-P" provided that "no S is P" is not also true' (op. cit., p. 281 n. 2, p. 201). Later: 'it appears that "every X is Y" is defined as equivalent to "no X is not-Y" *only when it is the case* that one of the two sub-contrary propositions, "some X is Y" or "some X is not-Y", is true' (p. 290). Otherwise, 'every S is P' is undefined. Sommers also holds analogously that 'if p then q' is undefined when 'p' is false (p. 321, n. 11).

[5] See A. N. Prior, *Formal Logic* (Oxford, $1962^2$), pp. 166–8; id., *The Doctrine of Propositions and Terms* (London, 1976), pp. 111–16.

propositions, AiB and AoB, retain their Aristotelian truth-conditions: AiB is true just in case some B is A, and AoB is true just in case some B fails to be A. But the universal forms are given non-Aristotelian interpretations: AaB is held true just in case nothing both is B and fails to be A, and AeB is held true just in case nothing both is B and is A. Thus for Brentano, AaB may be true in circumstances in which AiB is false. For if nothing at all is B, AiB is false (for no B is A), but AaB is true (for nothing both is B and fails to be A). The syllogistic which results from these specifications is, of course, sensibly different from Aristotle's: the moods *Darapti* and *Felapton*, for example, are not valid for Brentano.[1]

Given Brentano's interpretation, the Peripatetic reduction is not open to the objection raised in the last section. For **BaA** does not imply **BiA,** so that it is, *pro tanto*, a possible construe of 'If A, then B'. Indeed, it appears that wholly hypothetical syllogisms are generally reducible to Brentano's categorical syllogistic. Thus we might conclude that the Theophrastan reduction can after all be carried out—provided that its categorical basis is sufficiently un-Aristotelian.

## XXVI

As far as we know, Theophrastus did not attempt to apply to mixed hypothetical syllogisms the method of reduction which he applied to wholly hypothetical syllogisms. There, we may suppose, he was content with Aristotle's 'method of selection'. But it may seem desirable to extend the Theophrastan method to mixed cases; for if no extension is possible, the method will appear unpalatably *ad hoc*.

The extension places a further requirement on reduction: we need some way of translating *simple* propositions into categorical form. Different translations can be dreamed up. We might, for example, determine to construe the simple proposition P by way of **PiP.**[2] That is to say, 'P' is true just in case something which is such that P is such that P.[3] Similarly, not-P may be construed as

[1] See the formal discussion in Thom, op. cit. (p. 296 n. 1), pp. 111–13, where Brentano's system is an interpretation of Thom's $A^f$ or *B*. (But strictly speaking it is Thom's *BN*—see p. 121—which I advert to in the text: *BN* is *B* plus negative terms.)

[2] Sommers, op. cit. (p. 281 n. 2), p. 156, translates 'p' by 'a [p] obtains'; that could be represented as **OiP**, where O is the term *obtaining*.

[3] Since **BiA** is equivalent to 'A and B' (above, p. 317 n. 2), **PiP** is equivalent to 'P and P'—which is in turn equivalent to 'P'.

**PeP:** 'not-P' is true just in case nothing is both such that P and such that P.

*Modus ponens* inferences now assume the following form:

$$\frac{\begin{array}{l}\mathbf{BaA}\\ \mathbf{AiA}\end{array}}{\mathbf{BiB}}$$

And *modus tollens* emerges as:

$$\frac{\begin{array}{l}\mathbf{BaA}\\ \mathbf{BeB}\end{array}}{\mathbf{AeA}}$$

The second schema presents no difficulty: it is the categorical mood *Camestres*. The first schema is not itself a standard categorical mood; but it can be validated by way of the moods *Darii* and *Datisi*. From **BaA** and **AiA** infer **BiA** (by *Darii*); from **BaA** and **BiA,** infer **BiB** (by *Datisi*).

## XXVII

With ingenuity, further reductions of that sort can be effected. But it may well be wondered, at this point in the argument, what end such ingenuity might serve. Surely the end is no longer one of historical understanding; for it can hardly be supposed that the Brentanoesque manœuvres undertaken in the last sections have any direct historical application to the Peripatetic logicians. The later Peripatetics did not indeed follow Aristotle slavishly, and they modified his system at various points;[1] but they never considered a categorical syllogistic as un-Aristotelian as that of Brentano.[2] Nor did they evince any interest in extending Theophrastus' reductive techniques from wholly hypothetical to mixed hypothetical syllogisms.[3]

[1] Theophrastus, who modified Aristotle's categorical syllogistic in various ways, not all of them minor, did not question the Laws of Subalternation (see, for example, Alexander, *in A.Pr.* 69. 26–70. 21, and the other texts collected as Theophrastus, F 17 G: Theophrastus derives, for example, *Baralipton* from *Barbara* by converting the conclusion of *Barbara*, i.e. by inferring CiA from AaC).

[2] But see Thom, op. cit. (p. 296 n. 1), p. 128: 'Aristotle's logic of indefinite [i.e. negative] terms is . . . fragmentary. . . . The full system of which his system is a fragment is *BCN*', and hence a system with a Brentanoesque component. Perhaps we can say, after all, that the germs of a Brentanoesque system are present in Aristotle—though they did not sprout in the ancient Peripatetic tradition.

[3] This is not, I think, an accidental fact about the history of Peripatetic

If the further pursuit of reductions has no historical sense, it is presumably not without technical interest. Questions of reduction are, after all, at least semi-technical, and technical exercises have their own intrinsic value. But I believe that the debate between term logic and sentence logic involves more than purely technical issues.

Formal logic may be studied for a variety of purposes and under a variety of aspects. Under one perfectly respectable aspect, it may be seen as an attempt to systematize and to explain informal patterns of inference—in other words, as an attempt to codify the laws of thought.[1] There are several constraints on any such attempt. Formal logic must, for example, be *adequate* to ordinary inferential structures—it should, in principle, be capable of accounting for all the formally valid inferences which we normally recognize. It should also, I think, be *homogeneous*—that is to say, it should have a unified vocabulary, a cohesive set of rules, an organic structure: it should not be an aggregate of disjointed parts. Again, formal logic must be *natural*: it must reflect the underlying structure of ordinary discourse, and preserve and explain structural similarities and dissimilarities. These constraints are vague, perhaps essentially so; and they are flexible. But their general purport and their good sense are plain.

Sentence logic is often considered to do well on the score of adequacy and homogeneity, badly on the score of naturalness. Take, for example, the orthodox rendering in Fregean sentence logic of 'Every A is B'. The formula '$(\forall x)\ (Ax \supset Bx)$' imports a propositional connective into what is apparently a simple sentence. What is more, other sentences which apparently have the same structure as 'Every A is B' (for example, 'Most A's are B'), cannot be represented by any formula of the form '$(Qx)\ (Ax \supset Bx)$'.

Term logic, on the other hand, or at least its Aristotelian core, is often granted a considerable degree of naturalness. But it is now

logic, which was in a sense essentially piecemeal (see Barnes, op. cit. (p. 285 n. 4).

[1] There is nothing disreputable in the old notion that logic studies the laws of thought—provided that the notion is not mistaken for the thesis that formal logic is concerned with the ways in which men actually think, and provided, too, that we do not suppose such study to exhaust the scope of logical research. Aristotle was interested in codifying the laws of thought, but that was not his only reason for studying logic. The *Topics* show his interest in codification, the *Analytics* are, on the whole, less concerned with such matters: there, Aristotle's primary desire is to develop a formal system suitable for the presentation of scientific proofs, and his enterprise is analogous to Frege's in the *Begriffsschrift*.

generally regarded as inadequate—that, indeed, is the chief reason for the modern triumph of sentence logic. The Peripatetic programme was an attempt to show that, despite various difficulties, term logic is after all an adequate logic. If the ancient Peripatetic programme was doomed to failure, it may still be worth investigating the feasibility of a neo-Peripatetic programme—a programme unconstrained by the particular features of *Aristotelian* term logic. For the modern term logician's hope is to produce a system of logic which is adequate and homogeneous and which retains at least some of the naturalness of the Aristotelian core.[1]

Seen in this light, Theophrastus' attempt to reduce wholly hypothetical syllogisms to categorical syllogistic is of more than technical interest. For it is an essay in the general philosophical problem of codifying the laws of thought. The essay may ultimately prove futile, and term logic may eventually be laid to rest. Sentence logic may justify its recent rude usurpation. Or perhaps, after the thesis of term logic and the antithesis of sentence logic, we should, in Hegelian vein, expect a new synthetic logic.[2]

## APPENDIX

Alexander, *in A.Pr.* 325. 31–328. 7

*Ἐπιϲκέψαϲθαι δὲ δεῖ καὶ διελεῖν, ποϲαχῶϲ οἱ ἐξ ὑποθέϲεωϲ*

*Τοῦτο εἴρηκεν ἤτοι ὡϲ πάντων τῶν ἐξ ὑποθέϲεωϲ ὑποπίπτειν δυναμένων τῇ ἐκκειμένῃ τῶν ὅρων ἐκλογῇ τε καὶ τῇ δι' αὐτῶν δείξει (εἰ γὰρ ἐπιϲκέπτοιτό τιϲ καὶ διέλοι, εὑρήϲει τοῦτο οὕτωϲ ἔχον· εἰπὼν γὰρ τούϲ τε κατὰ μετάληψιν καὶ τοὺϲ κατὰ ποιότητα δεῖν φηϲιν ἐπιϲκέψαϲθαι καὶ τοὺϲ ἄλλουϲ τοὺϲ ἐξ ὑποθέϲεωϲ· ἐξ ὑποθέϲεωϲ γὰρ καὶ οἱ διαιρετικοί, οἳ καὶ αὐτοὶ ἐν τοῖϲ κατὰ μετάληψιν, ἐξ ὑποθέϲεωϲ καὶ οἱ ἐξ ὁμολογίαϲ· δεῖν οὖν φηϲι τῶν ἐξ ὑποθέϲεωϲ προϲεχεϲτέραν ποιήϲαϲθαι διαίρεϲιν), ἢ εἰπών, τίνεϲ τῶν ὑποθετικῶν φανερῶϲ ὑπάγονται τῇ ἐκκειμένῃ μεθόδῳ (οἵ τε γὰρ δι' ἀδυνάτου καὶ οἱ κατὰ μετάληψιν, ὑφ' οὓϲ πάντεϲ οἱ λεγόμενοι ἀναπόδεικτοι, καὶ ἔτι οἱ κατὰ ποιότητα), λέγει δεῖν ἐπιϲκέψαϲθαι*

[1] The extended Brentanoesque term logic, however, contains several unnatural features: **PiP** is an odd rendering of 'P'. Adequacy and naturalness are in tension, and a gain in one is often a loss in the other.

[2] Rough versions of parts of this paper were presented to seminars in Oxford, Göttingen, and London. I am grateful to my audiences for numerous helpful criticisms. I owe particular thanks to Pamela Huby, Antony Lloyd, Mario Mignucci, and Timothy Smiley.

καὶ διελεῖν, ποϲαχῶϲ οἱ ἐξ ὑποθέϲεωϲ λέγονται· ἐκ γὰρ τῆϲ διαιρέϲεωϲ δῆλον ἔϲται, εἴτε πάνταϲ οἷόν τε ὑπάγειν τῇ ἐκκειμένῃ μεθόδῳ οὔϲῃ δεικτικῇ εἴτε οὔ. δόξουϲι γὰρ οἱ δι' ὅλων ὑποθετικοί, οὓϲ Θεόφραϲτοϲ κατὰ ἀναλογίαν λέγει, οἷοί εἰϲιν οἱ διὰ τριῶν λεγόμενοι, μηκέτι ὑποπίπτειν τῇ διὰ τῆϲ ἐκλογῆϲ δείξει. λέγει δὲ αὐτοὺϲ ὁ Θεόφραϲτοϲ κατὰ ἀναλογίαν, ἐπειδὴ αἵ τε προτάϲειϲ ἀνάλογον καὶ τὸ ϲυμπέραϲμα ταῖϲ προτάϲεϲιν· ἐν πᾶϲι γὰρ αὐτοῖϲ ὁμοιότηϲ ἐϲτίν. ἢ οὐδὲ ϲυλλογιϲμοὶ κυρίωϲ καὶ ἁπλῶϲ ἐκεῖνοι, ἀλλὰ τὸ ὅλον τοῦτο ἐξ ὑποθέϲεωϲ ϲυλλογιϲμοί· οὐδὲν γὰρ εἶναι ἢ μὴ εἶναι δεικνύουϲιν. οἱ μὲν γὰρ προειρημένοι ἐξ ὑποθέϲεωϲ καὶ ϲυλλογιϲμοί· δεικνύουϲι γάρ τι ὑπάρχειν ἢ μὴ ὑπάρχειν· οἱ δὲ τοιοῦτοι μηδὲν τοιοῦτον δεικνύοντεϲ οὐκέτι οὐδὲ ἁπλῶϲ ϲυλλογιϲμοί. εἰ δὲ οὗτοι οὐδὲ τὴν ἀρχὴν ἁπλῶϲ ϲυλλογιϲμοί, πάντεϲ ἂν οἱ κυρίωϲ καὶ ἁπλῶϲ ὄντεϲ ϲυλλογιϲμοὶ διὰ τῆϲ προκειμένηϲ μεθόδου δεικνύοιντο.

Ἀνάγονται μέντοι καὶ οἱ δι' ὅλων ὑποθετικοὶ εἰϲ τὰ τρία τὰ προειρημένα ϲχήματα ἄλλῳ τρόπῳ, ὡϲ καὶ Θεόφραϲτοϲ δέδειχεν ἐν τῷ πρώτῳ τῶν Προτέρων ἀναλυτικῶν. ἔϲτι δὲ δι' ὅλων ὑποθετικὸϲ τοιοῦτοϲ· εἰ τὸ Α, τὸ Β, εἰ τὸ Β, τὸ Γ, εἰ ἄρα τὸ Α, τὸ Γ· τούτων γὰρ καὶ τὸ ϲυμπέραϲμα ὑποθετικόν· οἷον εἰ ἄνθρωπόϲ ἐϲτι, ζῷόν ἐϲτιν, εἰ ζῷόν ἐϲτιν, οὐϲία ἐϲτίν, εἰ ἄρα ἄνθρωπόϲ ἐϲτιν, οὐϲία ἐϲτίν. ἐπεὶ τοίνυν δεῖ καὶ ἐν τούτοιϲ μέϲον τινὰ ὅρον εἶναι, καθ' ὃν ϲυνάπτουϲιν αἱ προτάϲειϲ ἀλλήλαιϲ (ἄλλωϲ γὰρ ἀδύνατον καὶ ἐπὶ τούτων ϲυνακτικὴν ϲυζυγίαν γίνεϲθαι), οὗτοϲ ὁ μέϲοϲ τριχῶϲ καὶ ἐν ταῖϲ τοιαύταιϲ ϲυζυγίαιϲ τεθήϲεται. ὅταν μὲν γὰρ ἐν ᾗ μὲν τῶν προτάϲεων λήγῃ, ἐν ᾗ δὲ ἄρχηται, τὸ πρῶτον ἔϲται ϲχῆμα· οὕτωϲ γὰρ ἕξει, ὡϲ καὶ ὅτε τοῦ μὲν τῶν ἄκρων κατηγορεῖτο, τῷ δὲ ὑπέκειτο. ἀνάλογον γὰρ τὸ μὲν λήγειν καὶ ἕπεϲθαι τῷ κατηγορεῖϲθαι, τὸ δὲ ἄρχεϲθαι τῷ ὑποκεῖϲθαι· ὑπόκειται γάρ πωϲ τῷ ἐπιφερομένῳ αὐτῷ. οὕτωϲ γὰρ ληφθέντοϲ τοῦ μέϲου ϲυμπέραϲμα ἔϲται, ὃ ἄρχεται μὲν ἀφ' οὗ ἤρχετο καὶ ἡ πρώτη πρόταϲιϲ, λήγει δὲ εἰϲ ὃ ἔληγεν ἡ δευτέρα, τὴν μὲν τοῦ κατηγορουμένου χώραν ἐν τῷ ϲυμπεράϲματι τοῦ ἑπομένου λαμβάνοντοϲ τὴν δὲ τοῦ ὑποκειμένου τοῦ ἡγομένου· οἷον εἰ τὸ Α, τὸ Β, εἰ τὸ Β, τὸ Γ, εἰ ἄρα τὸ Α, τὸ Γ. δύναται ἐπὶ τῇ τοιαύτῃ ϲυζυγίᾳ καὶ ἀνάπαλιν ληφθῆναι τὸ ϲυμπέραϲμα ὥϲτε μὴ ἑπόμενον εἶναι ⟨τὸ Γ⟩ ἀλλ' ἡγούμενον, οὐ μὴν ἁπλῶϲ
327. ἀλλὰ ϲὺν ἀντιθέϲει· ϲυναχθέντοϲ γὰρ τοῦ εἰ τὸ Α, τὸ Γ ϲυνάγεται καὶ τὸ εἰ μὴ τὸ Γ, οὐ τὸ Α. εἰ δὲ ἀπὸ διαφόρων ἀρχόμεναι αἱ ὑποθετικαὶ προτάϲειϲ λήγοιεν εἰϲ ταὐτό, ἔϲται τὸ τοιοῦτον ϲχῆμα δεύτερον ἀνάλογον ὂν τῷ ἐν τοῖϲ κατηγορικοῖϲ δευτέρῳ, ἐν οἷϲ ὁ μέϲοϲ ὅροϲ ἀμφοτέρων τῶν ἄκρων κατηγορεῖτο· ἐπεὶ γὰρ ἐν τοῖϲ ὑποθετικοῖϲ τὸ ἑπόμενον κατηγορουμένου χώραν ἔχει, ὅταν ἐν ταῖϲ δύο προτάϲεϲι ταὐτὸν ἑπόμενον λαμβάνηται, τὸ δεύτερον ἔϲται ϲχῆμα. ϲυλλογιϲτικὴ δὲ ἡ ϲυζυγία ἂν ἀντικειμένωϲ ἑπόμενον ἑκατέρῳ τῶν ἡγουμένων λαμβάνηται, οἷον εἰ τὸ Α, τὸ Γ, εἰ τὸ Β, οὐ τὸ Γ· τὸ γὰρ Γ μέϲοϲ ὢν ὅροϲ ἀντικειμένωϲ εἴληπται ἑπόμενοϲ τοῖϲ ἡγουμένοιϲ, τῷ τε Α καὶ τῷ Β. διὸ καὶ ϲυναχθήϲεται οὕτωϲ ληφθέντων τὸ εἰ θάτερον τῶν ἀρχομένων, οὐ θάτερον· εἰ γὰρ τὸ Α, τὸ Γ, εἰ

**326.** 38 ⟨τὸ Γ⟩ addidi. **327.** 11 τὸ Γ (Prantl)] τὸ Β codd.

τὸ Β, οὐ τὸ Γ, εἰ ἄρα τὸ Α, οὐ τὸ Β, οἷον εἰ ἄνθρωπος, ζῷον, εἰ λίθος, οὐ ζῷον, εἰ ἄρα ἄνθρωπος, οὐ λίθος. εἰ δέ γε ἀπὸ τοῦ αὐτοῦ ἀρχόμεναι αἱ προτάσεις λήγοιεν εἰς ἕτερα, ἔσται ἀνάλογον τοῦτο τὸ σχῆμα τῷ τρίτῳ· τὸ γὰρ ἡγούμενον ὑποκειμένου χώραν ἔχον ἐν ἀμφοτέραις ταῖς προτάσεσι ταὐτόν ἐστιν. ὅταν δὴ ἀντικειμένως τοῦτο ληφθῇ, συνακτικὸν ἔσται, οἷον εἰ τὸ Α, τὸ Β, εἰ οὐ τὸ Α, τὸ Γ· συναχθήσεται γὰρ εἰ μὴ θάτερον τῶν ληγόντων θάτερον· εἰ γὰρ οὐ τὸ Β, τὸ Γ, ἢ εἰ οὐ τὸ Γ, τὸ Β, οἷον εἰ ἄνθρωπος, λογικόν, εἰ μὴ ἄνθρωπος, ἄλογον, εἰ μὴ λογικὸν ἄρα, ἄλογον. ταύτῃ τε οὖν ὅμοιαι αἱ ἐν τούτοις συμπλοκαὶ ταῖς ἐν τοῖς κατηγορικοῖς σχήμασιν οὖσαι εἰκότως ἂν εἰς ἐκείνας ἀνάγοιντο. καὶ ἔτι ἡ γένεσις ὥσπερ ἐν τοῖς κατηγορικοῖς τῷ δευτέρῳ καὶ τρίτῳ σχήματι ἀπὸ τῶν ἀντιστροφῶν τῶν ἐν τῷ πρώτῳ προτάσεων, οὕτως δὲ καὶ ἐν τούτοις· τῆς μὲν γὰρ μείζονος ἀντιστραφείσης ἐν πρώτῳ σχήματι προτάσεως τὸ δεύτερον ἐγένετο σχῆμα, τῆς δὲ ἐλάττονος τὸ τρίτον. ἔστι δὲ τοῖς ὑποθετικοῖς μείζων μὲν ἡ δευτέρα, ἐν ᾗ ἡγεῖται ὁ μέσος, ἐλάττων δὲ ἡ πρώτη, ἐν ᾗ ἕπεται ὁ μέσος· οἷον ἡ μὲν εἰ τὸ Α, τὸ Β πρώτη τε καὶ ἐλάττων, ἡ δὲ εἰ τὸ Β, τὸ Γ δευτέρα τε καὶ μείζων. τῆς μὲν οὖν εἰ τὸ Β, τὸ Γ ἀντιστραφείσης ἔσται ἐν ἀμφοτέραις τὸ Β ἑπόμενον καὶ τὴν χώραν λαμβάνον τοῦ κατηγορουμένου, ὃ ἴδιον τοῦ δευτέρου σχήματος· τῆς δὲ πρώτης ⟨τῆς⟩ εἰ τὸ Α, τὸ Β ἀντιστραφείσης ἔσται πάλιν ἡγούμενον ἐν ἀμφοτέραις ταῖς προτάσεσι τὸ Β, ὃ χώραν ὑποκειμένου ἔχον ποιεῖ τὸ τρίτον σχῆμα. παραπλησίως δὲ καὶ αἱ ἀναλύσεις τῶν ἐν τῷ δευτέρῳ καὶ τρίτῳ σχήματι ἐς τὸ πρῶτον ἔσονται σχῆμα, ἐξ οὗ καὶ αἱ γενέσεις αὐτοῖς, ὥσπερ καὶ ἐπὶ τῶν κατηγορικῶν. οὗτοι μὲν οὖν οἱ ἁπλοῖ τε καὶ πρῶτοι ὑποθετικοὶ δι' ὅλων λεγόμενοι. ἐκ τούτων δὲ καὶ οἱ σύνθετοι πάντες τὴν σύστασιν **328.** 1 ἔχοντες δειχθήσονται. Θεόφραστος μέντοι ἐν τῷ προτέρῳ τῶν Ἀναλυτικῶν δεύτερον σχῆμα λέγει ἐν τοῖς δι' ὅλων ὑποθετικοῖς εἶναι ἐν ᾧ ἀρχόμεναι ἀπὸ τοῦ αὐτοῦ αἱ προτάσεις λήγουσιν εἰς ἕτερα, τρίτον δὲ ἐν ᾧ ἀπὸ διαφόρων ἀρχόμεναι λήγουσιν εἰς ταὐτόν. ἀνάπαλιν δ' ἡμεῖς ἐξεθέμεθα. ἀλλὰ περὶ μὲν τούτων ἰδίᾳ καιρὸς ἂν εἴη λέγειν. νῦν δ' ἐπανιτέον ἐπὶ τὴν τῆς λέξεως ἐξήγησιν.

**327.** 12 τὸ Β, οὐ τὸ Γ] τὸ Γ, οὐ τὸ Β codd.

### We must investigate and distinguish the number of ways in which arguments from a hypothesis . . . [*A.Pr.* $45^{b}19$]

He says this either supposing that all arguments from a hypothesis are amenable to the selection of terms in question and to proof by way of them (if / you investigate and distinguish, you will find that this is so; for **325.** 35 having spoken of arguments 'in virtue of a changed assumption' and those 'by virtue of a quality', he says that we must also investigate the other arguments from a hypothesis—for 'separative' arguments, which are / themselves included among those 'in virtue of a changed **326.** 1 assumption', are from a hypothesis, and so too are arguments from an agreement: so he is saying that we should make a more careful distinction among arguments from a hypothesis); or else, having said which hypothetical arguments are evidently brought under the method in question / (*per impossibile* arguments, and arguments 'in virtue of a changed assumption', which include all the so-called 'indemonstrables'—and also arguments 'in virtue of a quality'), he means that we must investigate and distinguish the number of ways in which arguments are said to be from a hypothesis—for it will be plain from the distinction whether or not it is possible to bring all of them under the method in question, which is a method of proof. For wholly hypothetical arguments, which Theophrastus calls arguments 'in virtue of an analogy',—e.g. the so-called 'three component' arguments—will be thought not to be / amenable to proof by selection. (Theophrastus calls them arguments 'in virtue of an analogy' because the premisses are analogous to each other and the conclusion to the premisses—they all show a similarity.)

Perhaps these arguments are not syllogisms in the proper sense and without qualification, but rather syllogisms-from-a-hypothesis (the phrase being taken as a whole). For they do not prove that anything is or is not the case. The arguments / from a hypothesis mentioned earlier are indeed syllogisms (they prove that something holds or does not hold), but these do not prove anything of that sort and so are not syllogisms without qualification. And if they are *not* syllogisms without qualification, then *all* arguments which are syllogisms in the proper sense and without qualification are proved by the method in question.

However, wholly hypothetical arguments also reduce, in another way, to the three figures mentioned earlier, as Theophrastus has proved in the first book of his *Prior Analytics*.

An argument of the following sort is wholly hypothetical:

If A, then B
If B, then C
Therefore: if A, then C.

(Here the conclusion too is hypothetical.) E.g.:

If he is a man, he is an animal
If he is an animal, he is a substance
Therefore: if he is a man, he is a substance.

Now since here too there must be some middle term in virtue of which the premisses connect with one another (for if not, here too it is impossible for there to be any concludent pairing), this middle term will be positioned in three ways in pairings of this sort too.

When it is consequent in one of the premisses and antecedent in the other, we will have the first figure. / For then it will be in the same case as when it is predicated of one of the extremes and is subject for the other. For being a consequent or apodosis is analogous to being predicated, and being antecedent to being subject—for in a way it is subject for what is inferred from it. For when the middle term is taken in this way there will be a conclusion the antecedent of which was also the antecedent of the first premiss and the consequent the consequent of the second—the apodosis / taking the place of the predicate in the conclusion and the antecedent that of the subject. E.g.:

If A, then B
If B, then C
Therefore: if A, then C.

In the case of pairings of this sort the conclusion can also be taken the other way about, so that ⟨C⟩ is not the apodosis but the antecedent—the other way about not without qualification / but including negation. For once 'If A, then C' is concluded, so too is 'If not-C, then not-A'.

If the hypothetical premisses have different antecedents and the same consequents, this sort of figure will be the second, being analogous to the second figure in categorical arguments in which the middle term / is predicated of both the extremes. For since in hypothetical arguments the apodosis holds the place of a predicate, then whenever the same consequent is taken in the two premisses, we will have the second figure. The pairing is syllogistical if the apodosis of the two antecedents is taken in contradictory fashion— e.g.:

If A, then C
If B, then not-C.

For C, the middle term, is taken in contradictory fashion as apodosis of / the antecedents A and B. Hence if they are taken in this way it will be concluded that if one of the antecedents holds the other does not:

If A, then C
If B, then not-C
Therefore: if A, then not B.

E.g.:

If a man, then an animal
If a stone, then not an animal
Therefore: if a man, then not a stone.

If the premisses have the same antecedents and different consequents, this figure will be analogous to the third; / for the antecedent, which holds the place of a subject, is the same in both premisses. Now when this is taken in contradictory fashion, it will be concludent. E.g.:

If A, then B
If not-A, then C.

For it will be concluded that if one of the consequents does not hold, the other does (if not-B, then C; or if not-C, then B). E.g.:

If a man, then rational
If not a man, then non-rational
Therefore: if not rational, then / non-rational.

Thus the combinations in these arguments, being similar in this way to those in the categorical figures, should reasonably reduce to them. Moreover, just as with categoricals the second and third figures are generated from the converses of the premisses of the first, so it is with these arguments too. For when the major premiss in the first figure was converted, the second / figure was generated, and when the minor the third. In hypotheticals the second premiss, in which the middle term is the antecedent, is the major, and the first, in which the middle term is the apodosis, is the minor. E.g. 'If A, then B' is first and minor, 'If B, then C' second and major. So when 'If B, then C' is converted, in both premisses B will be the apodosis and take / the place of the predicate—and that is the defining mark of the second figure. And again, when the first premiss, 'If A, then B', is converted, B will be the antecedent in both premisses—and, holding the place of the subject, it produces the third figure.

Similarly, arguments in the second and third figures will also be analysed into the first figure, from which they are generated—as / in the case of the categoricals too.

328. These, then, are the simple and primary / so-called wholly hypothetical arguments. All the compound wholly hypothetical arguments will be proved to be constituted from them.

Theophrastus, however, says in the first book of his *Analytics* that the *second* figure in wholly hypothetical arguments is the one in which the premisses have the same antecedent and different consequents, and the *third* that in which they have / different antecedents and the same consequent. We have set them out the other way about. There may be an occasion to discuss these issues separately: for the present we must return to the elucidation of the text.