On Higher-Order Logic and Natural Language

JAMES HIGGINBOTHAM

1. Introduction

I WILL EXPLORE some considerations on behalf of, and against, second- and higher-order logic that take for at least part of their motivation the properties of natural languages. A couple of preliminary remarks are appropriate here.

There is first of all the question whether second-order logic really is logic. Suppose that logic is understood in a traditional way, as the most general theory of the true and the false, abstracting from the subject matter of the special sciences but applicable to all of them. The traditional conception certainly includes all of the logic of the truth functions—that much of logic arises as soon as we have distinguished truth from falsehood—but it is not altogether trivial to arrive at the conclusion that even first-order logic is logic. The reason is that, besides truth and falsehood, first-order logic requires the additional notion of satisfaction, or a predicate's being true *of* an object; hence two additional concepts, that of truth of, and that of an object. But if all of first-order logic is admitted as logic, second-order logic appears at first to require at most only one further additional concept, namely that of the value of a predicate variable. Supposing this concept acceptable, second-order logic would have the required degree of generality and topic-neutrality: it is not biased in such a way as to exclude (except, perhaps, by failing to contain enough logical resources) any special science from being expounded in a language of which it is the logic. Thus I will suppose that if the notions

additional to second-order logic can be motivated, second-order logic qualifies as logic in at least as strict a sense as first-order logic with identity; and once second-order logic is admitted, further extensions are not only possible but would even appear inevitable.

Preliminary notice must also be taken of the concept of 'natural language' that will be at stake in what follows. A natural language, as I will understand it, is not a historically given, as opposed to an artfully or artificially created language, but rather a language that is natively available to us for acquisition and use as a first language, under normal environmental conditions. Many natural languages in this sense of the term are no longer spoken, some will be spoken but have not yet been, and others will never be spoken by anyone. To look at the matter platonistically, natural languages are a proper subset of possible languages, and a proper superset of existing human first languages.

If natural languages are identified with historically given modes of human communication, they will include mechanisms that do not figure in human first languages at all, an obvious example being the use of small Roman and Greek letters to mark places of quantification. Of course, these and similar pieces of mathematical and other general scientific idioms are in part artificial, having been, like musical notation, self-consciously created. But it would be wrong to regard them as wholly artificial to *us*, who stand downstream from refinements and adjustments by many people over many years. If we think of our language as taking these historical elements on board, on the ground that we are educated to use them freely, then we shall have obliterated the distinction that I wish to make, between motivations for second-order logic that can be recovered from the design of human first languages and motivations that stem from our general practices of thinking, speaking and writing. From the perspective I am taking, however, it is possible that a good case for second-order logic can be made out by appealing to the additional practice, even if it cannot be launched entirely from our clarification of the semantics of the most basic parts of language.

Much of the literature on the problem of defending or advancing higher-order logic concentrates first of all on the case for adding monadic predicate variables in predicate position, thus not including predicate variables of more than one place or even monadic predicates as arguments. I will follow this practice for the most part, noting however that if extensions beyond the monadic case are more difficult to motivate the reasons may be essential or adventitious; that is, they may

reflect limitations on the expressive power of natural languages, or what we might call syntactic accidents that preclude the well-formedness of what would otherwise be the most intuitive constructions. Our questions are naturally related to questions about the motivation for property- and relation-abstraction from within natural language, and whether properties and relations should be taken as essentially predicational entities or as objects of some sort. Here and below, I use 'predicational' for 'having the value that a predicate has' as opposed to 'having the value that a singular term has' (of course, in some views these cannot be distinguished); and I also distinguish 'predicational' from 'predicative', reserving the latter for definitions, or things definable in terms of definitions, that are mathematically predicative; i.e., do not involve quantification over domains that include the thing being defined.

2.   Elementary linguistic motivations

Contemporary native speakers of the language in which this article is written will have been introduced to the explicit treatment of generality by means of the systematic discussion of the expressions 'all' and 'some' and related words, appearing in construction with possibly complex nominals. Not only these, but also the quantificational adverbs 'always' and 'sometimes' and related words, can be used for the purpose, and in some human languages these are far more common than the nominal quantifiers. Abstracting from the grammatical difference between 'All men are mortal' and Russell's 'Men are always mortal', we have the restricted quantification 'for every x such that x is a man, x is mortal' or the equivalent unrestricted version 'for every x, if x is a man then x is mortal'. Now we find these quantifiers, as well as (analogues of) the non-standard quantifiers, only some of which are first-order definable, also in predicate position, as in

(1)   John is everything we wanted him to be

We also find inferences with predicational quantifiers paralleling inferences with objectual quantifiers, as in

(2)   John is mostly what we expected him to be;
      The only things we expected John to be are: honest, polite, and scholarly:
      Therefore, John is either honest or polite.

A systematic treatment of these inferences will call for explicit variable-binding, but this time of predicate positions, on a par with ordinary quantification over the argument positions of predicates.

Examples like (1) and (2), however, do not show much. The inference in (2) would readily be graspable, and correct, whether the quantification were truly second-order or merely projected by analogy onto the category of adjective phrases from the argument categories, and so in particular if it were substitutional. The quantifier words are generally restricted to certain constructions (as we had to use the dummy noun 'thing' in the second premise of (2)), but the *wh*-expressions, which function at least in a quasi-quantificational manner in questions, range over more categories, including prepositional phrases of manner ('How did John fix the car?' 'With a wrench'), quantificational adverbs ('How often does John walk to work?' 'Rarely'), and others. These will correspond to quantifications using dummy nouns when the questions are embedded and numerical words are added. So we have

    (3)   I know three ways John could fix the car
    (4)   Mary wondered with what frequency John walked to work

and so forth. Once we are launched into a second-order semantic framework for natural language, we may (and many do, following especially the lead of Richard Montague) take these constructions as exhibiting the possibility of ascending to higher simple types. Thus if prepositional phrases are adverbials in the sense of categorial grammar, belonging to a category that makes one-place predicates from one-place predicates, then in the extensional setting their model-theoretic semantics will have them taking as values functions from sets of individuals to sets of individuals; and if expressions like 'rarely', 'more often than Bill does' and the like are adverbs of quantification, having for their values sets of individuals, then abstraction over such adverbial prepositional phrases as 'with what frequency' in (4) will give predicates true of sets of sets of individuals. But since we are concerned with the question whether any ostensible form of quantification over other than argument positions of ordinary predicates (nouns, verbs, adjectives, and prepositions) can motivate the higher-order perspective in the first place in anything like the serious way reflected in the type-theoretic hierarchy, we have as yet no reason to subscribe to the extension.

It is a natural conjecture that in the first instance quantification over other than ordinary argument positions is substitutional. Of course we

intend these quantifications, say over positions of one-place predicates, not to be restricted to the substitution class that we actually have available at this stage of our language, but also to include all predicates possible for us, and perhaps coextensive with actual predicates for speakers of other languages. The point would count against the substitutional treatment if we were thinking of *defining* truth for statements of the class. It is analogous to the objection that if reference, satisfaction, and truth are defined in the familiar Tarskian manner, then we are deprived of saying the natural things about language change. In particular, we lose the distinction between replacing one language by another, and merely extending a language (see Field 1972). If what we want to do, however, is simply to use the truth conditions of statements as part of the clarification of their meaning, then nothing prevents us from saying that we intend, as new predicates are added to our language, to enlarge the substitution class so as to include them. We think of ourselves as prepared to 'go on as before' and we can even explain what going on as before means to us. A similar response is available to the worry that if logical laws are stated by means of 'semantic ascent' rather than higher-order quantification, the theorist is prisoner of whatever language is in question at the time; though in this case one may note that the laws that hold in our language will provably hold also in extensions of it. Hilary Putnam once argued that when one asserted the validity of $(\forall x)(F(x) \lor \neg F(x))$ and the like, one was 'implicitly making second-order assertions' (Putnam 1971, p. 31)). But a demonstration is lacking that we must intend by the assertion more than, 'No matter what may be put for F, $(\forall x)(F(x) \lor \neg F(x))$ is true.' On the other hand, to have escaped the problem of the values of predicate variables and their intended range by going substitutional yields only a very weak theory. (See Parsons 1971 for a predicative theory of classes along substitutional lines.)

If there are linguistic considerations in favour of second-order logic—examples such as (1) and (2), and the suspicion that the substitutional interpretation may not do justice to what we intend—there are also linguistic considerations against it. Whereas it is easy to construct ordinary English (or other natural language) sentences instantiating quite arbitrary schemata of first-order logic, the second-order instances that present themselves seem to be very restricted in their nature, generally speaking taking the form $[QX] \ldots X \ldots$, where Q is a quantifier and replacement of X by a predicate constant would give

merely a first-order schema. A particular difficulty that presents itself is that of giving second-order definite descriptions

     (5)   the X such that . . . X . . .

in subject position, where the description is rounded off by a common noun in construction with a relative clause. The problem is partly that significant common nouns are true of objects, so one must use a dummy noun such as 'thing' in place of a substantive head, and partly that ordinary predicates are likewise true of objects. Notoriously, Frege, having stated that the reference of a predicate-expression was a concept, found that one could not felicitously use this noun as a predicate of a definite description of a concept. Frege laid the problem to a difficulty 'in which language finds itself,' urged that at any rate the problem did not apply within *Begriffsschrift*, and laid the difficulty to the definite article *the* (or *der*) which 'points to an object.' As I have argued elsewhere (1989 and 1990), Frege appears to have been mistaken in the last assertion, since the definite article (like any quantifier) can function perfectly well in construction with second-order variables. The difficulty, rather, is that ordinary predicates demand *saturated* arguments, whereas the definite description schema (5) above is unsaturated. Being unsaturated, it has instances that can function as predicates, as in

     (6)   John is [the very thing we expected him to become]

The trouble is that these descriptions are not in general acceptable as arguments. One can, however, agree with Frege otherwise: there is no difficulty in predicating of concepts in *Begriffsschrift*, and Frege would be justified from his own point of view in taking as adventitious the design feature of natural language that forces arguments generally to be saturated. I conclude, therefore, that second-order logic cannot just be written off on the score of such limitations.

    The above discussion, if correct, leaves the problem of motivating or dismissing the promotion of second-order logic within natural language almost exactly where it was. To promote second-order logic requires showing that a substitutional interpretation of apparent quantification over predicate positions is too weak, and that the limitations in constructing instances of second-order schemata are adventitious. On the other hand, second-order logic cannot be dismissed out of hand.

3. *Values of variables*

I said above that second-order logic would qualify as logic if one could get over the hump of identifying the values of predicate variables in an appropriate way, legitimating the additional concept. It has often been suggested that the additional concept puts second-order logic at least out of logic if not out of court. The latter conclusion would follow from a harsh reading of Quine's notorious slogan, that second-order logic is 'set theory in sheep's clothing' (Quine 1970, p. 66). A classic rebuttal, due to George Boolos (1975), is that there is simply no necessity to see the values of predicate variables (or constants) as sets or classes in disguise; no necessity, that is, to see them as surrogate names. Quine's objection can recur, however, in derived form. Thus Jody Azzouni asks in recent work how if at all second-order theories with standard models differ from certain first-order translations of them in terms of classes and membership, where the translation is accompanied by a simultaneous restriction on the class of models. He writes that

> the notation '*Pa*', which is a one-place predicate symbol syntactically concatenated with a constant symbol, is not taken to contain an (implicit) representation of $\in$. However, as soon as we allow ourselves to quantify (standardly) into the predicate position, this is precisely how syntactic concatenation *must* be understood. Furthermore . . . syntactic concatenation in these contexts is *not* open to reinterpretation across models—it is an (implicit) logical constant. (Azzouni 1994, p. 17)

The concept of concatenation figures in syntax, since the clauses that build formulas use it. If it has any semantic interpretation, that interpretation is uniform across models. But it does not follow that concatenation has any semantic interpretation. Azzouni's objection, however, is that the notion of a standard model *for* a second-order language stipulates that the monadic predicate variables are to range over *all* families of objects of the individual universe of discourse. (Here and below I use 'family of objects' as neutral among various conceptions of the kinds of values predicate variables may be said to have.) The stipulation is essential, because second-order logic is not recursively axiomatizable, and for recursively axiomatized fragments of it there will be a class of 'generalized' models of the type due to Leon Henkin (1950), such that exactly the theorems of the fragment are true in every model of the class; these models will not be standard. Confined as we may be said to be to recursively axiomatized systems, we cannot

implicate as it were within the second-order language that the models to be considered for evaluating logical truth are just the standard ones. Now, we could in a way implicate this if we took concatenation as suppressing a silent $\in$, viewed as a logical constant, and took the predicate variable as ranging, for each universe of discourse, over all of its subsets. But when with this understanding we write $(\exists F)F(a)$ we invisibly posit a uniform relation between the values that the predicate variable F may take on and the reference of a. The consequence Azzouni draws is a version of Quine's thesis that second-order logic is set theory in sheep's clothing, with the modification that the wolf revealed when the clothing is stripped away is not set theory, but a two-sorted first-order theory of objects and their classes.

Thus far I have been following Azzouni's text and to some extent interpreting it. In connection with our present concerns, the conclusion would be that insofar as we implicate the understanding that we are supposed to have of the range of our predicate variables, the value of a predicate variable is an object. Of course, the implication cannot always be forthcoming. Even if the values of the ostensible predicate variables are classes, we do not implicate the standard interpretation of our discourse except where the universe of individuals constitutes a set. But if we can take it as a set, then it falls short of what we intend for set theory, viz. that the quantifiers range over all sets. In any case, Azzouni's argument assumes that the set-theoretic models for second-order languages, and the interpretations of those languages on the other hand, assign the same sorts of things as values of predicate variables, or at least that there is no significant difference between them. But from within at least one perspective on second-order languages we need not look at matters this way. We can instead regard the model theory as merely giving a picture, within a first-order language, of what is intended, and deny that the values of predicate variables are objects, construing them instead as Frege did, as essentially predicational.

To specify an interpretation for a quantificational language we need to specify the range of quantification over individuals (which may, indeed, be everything). But then for second-order languages as well as first-order ones it seems that there is nothing *more* to be specified about quantification; that is, the range of the monadic predicate variables is itself determined by the range of the objectual variables. Consider this point from a Fregean perspective. As you know, Frege held that predicate-expressions—what was left over when one or more occurrences of a proper name were identified in a complete sentence

and the sentence was viewed as constructed from that name and the residue left upon removal of those occurrences—referred to 'unsaturated' things which he called concepts. Frege took it as a matter of importance that the concept, being unsaturated, did not in order to deliver a truth value when given an object require any relation between the concept and the object at all; rather, the concept was itself of such a nature as to have this property. Because of his doctrine that concepts were a special case of functions, Frege's way of setting things up does not exactly correspond to ours. Still, the unsaturatedness of the values of predicate variables does play at least a negative role in responding to the suspicion that second-order quantification is really quantification over classes of individuals, in that we need not explain why the concatenation of a monadic predicate variable with a proper name yields a syntactic object that has a truth value on assignment of a value to the variable. Moreover, the realm of concepts will comprise all ways of discriminating among objects; that is, the range is simply all concepts.

The last paragraph expresses a point of view toward second-order quantification that Stewart Shapiro (1990) calls *neutral realism*. Neutral realism is not undermined by Azzouni's objection, because it is by no means committed to the thesis that we understand second-order quantification in terms of membership in classes (though that is not to say that we can refute someone who chooses to understand us in this way). The objection invites one to defend the thesis that the second-order quantifiers range over all—really, all—concepts, by specifying intended models in terms of set theory. But that is an adventure that need not be undertaken. There is no question of interpreting, or reinterpreting, concatenation. Concatenation expresses nothing, serving only to indicate what is being predicated of what.

It is intrinsic to Frege's conception of second-order logic, however, that it at once invites us to ascend further. Once we have, say, $F(a)$, with proper name 'a' and variable F, we have the notion of a concept under which a falls, something that discriminates among concepts: some concepts are concepts under which a falls, others are not. The way is then open to concepts of Frege's second level, and so on up. Now the unrestricted quantifiers, as Frege showed, refer on this scheme to concepts of the second level. In natural language quantifiers are generally, and perhaps without exception, restricted (but Frege noticed this point too, remarking that in language quantifiers referred to relations between concepts: 1892, p. 48). Beyond the level of relations between concepts, it requires argument to show that natural language has any serious

means of expression at all. This or any other limitation on natural language, however, does not limit the hierarchy, which will ascend inevitably to a logic of order ω. Why then does our language not do the same, if the theory is correct?

Boolos (1984) proposed to define truth for second-order languages within second-order metalanguages without making classes, or anything else, count as the values of predicate variables. The definition uses quantification over relations $R(W, x)$ where x ranges over objects and W over monadic second-order variables, or, given a pairing function, predicates $R(<W,x>)$ over ordered pairs of variables and objects. On either conception, if V is such a variable, then each object o such that $R(V,o)$ is a value of V according to R. The class of such o is the extension of V according to R; but the extension need not be brought in for the purpose of defining truth, so there is no need to think of anything as being *the* value of V according to R. Thus one might conclude that after all the notions of objecthood and truth-of suffice for the semantics of at least second-order languages with only monadic predicate variables. Parsons (1990, p. 327–8) proposes a difficulty, which I interpret as follows. What is defined inductively is a relation 'R and s satisfy Φ', where Φ is a formula, s is an ordinary infinite sequence of objects and R is as above. But *that* notion is expressed by a triadic predicate with a second-order argument, namely R. If such predicates are admitted, we may consider in particular the predicate true of X if and only if X is $\lambda x.R(V,x)$ for some R and some V. That predicate is: X is a value of a second-order variable. Moreover, that X is *the* value of V according to R is: $X = $ (the Y)($Y(x) \leftrightarrow R(V,x)$). The values of second-order variables are in this way reconstructible within the system proposed.

In virtue of the above considerations, I will henceforth be assuming that to motivate second-order logic it must be shown that apparent second-order variables have values, and that these values are not objects, but rather things of an essentially predicational character. The latter statement responds to Azzouni's version of the sheep's-clothing charge, whereas acquiescence in some conception of the value of a predicate variable appears, in view of the difficulty Parsons brings to bear for Boolos's construction, to be required for second-order semantics. The issues in the sections following will concern the extent to which the Fregean conception can be supported within natural language. I conclude this section with a discussion of one attempt to locate within natural language higher-order logic as a basis for arithmetic.

Harold Hodes (1984 and further in his 1990), takes what appears to be at least very close to a Fregean realist position with respect to higher-order quantification, supporting it in part by reference to elementary examples (1990, p. 254–5). He does not elaborate reasons for supposing that these examples are as robust as the Fregean view would make them, but his conception does bring to the fore the question what to say about the numerals and other expressions designating cardinalities, when they are used either as predicates in their own right or as parts of complex quantifiers, as in (7)–(8) and the like:

(7)   The grains of sand on the beach are more than you can count

(8)   There are exactly four people in the living room

In the first-order arithmetic, supplemented with empirical predicates and classes, these statements go over as (9)–(10) respectively

(9)   For each n, if n is a number you can reach by counting, the number m of $\{x: x$ is a grain of sand on the beach$\}$ exceeds n

(10)   the number n of $\{x: x$ is a person in the living room$\}$ = 4

Hodes suggests that the number-words should be taken as predicates of the second level; i.e., predicates of concepts. Now, (10) is paraphrasable by a first-order statement using only the predicate '( ) is a person in the living room' plus quantification and identity. In Hodes's view that paraphrase in fact expresses the same *thought* as (10) (in a sense of that term intended to echo Frege), whose fundamental logical form would show '4' as predicate of '( ) is a person in the living room'. The numbers then, what the numerals refer to, are construed as concepts of the second level.

The development of arithmetic within such a framework requires further ascent up the hierarchy of levels since, e.g., predicates of functions such as '( ) is recursive' will refer to concepts of the fourth level. Moreover, the number-words must have something like Russell's 'typical ambiguity', since anything in the hierarchy can be counted. Numbers themselves can be counted, and so a statement like 'There are four prime numbers between 7 and 21' would, for numbers n at any level, involve a number of the next level up. In addition, the axiom of infinity must be assumed. But what of the thesis that (10) and what we may write as

(11)   4(( ) is a person in the living room)

express the same thought? The thesis must be not that (10) and (11), with their different structures, express the same thought, but rather that (10) is a potentially misleading expression of the thought whose form is transparent in (11); and similarly for the more strictly Fregean versions of (10) (involving value-ranges rather than sets). Hodes thinks otherwise (1984, p. 129), or so I read him. But if thoughts are the reference of complete sentences indicating the contents of belief and other psychological or epistemic states, their individuation must be finer than Hodes allows. Knowing as I might that there are distinct persons x, y, z and w in the living room and everyone in the living room is identical to x or y or z or w, I may not know there are exactly four persons in the living room. When I know that, I know that the persons in the living room are to be enumerated like this: one, two, three, four, and that's all. At the very least, Hodes's conception makes it too easy to have arithmetical knowledge (on this point see also Dummett 1991, chapter 14). It also seems to make it too easy to have knowledge of the content of complex sentences such as 'there are distinct persons x, y, z and w . . . ' etc, inasmuch as putting objects into one-to-one correspondence with an initial segment of the numerals (i.e. counting) is a very different and often easier task than grasping long statements with identity and multiple quantifiers.

4. *Properties and relations*

English and other languages have fully productive (modulo some syntactic details) means of nominalization, which will turn any open sentence $\Phi(x_1, \ldots, x_n)$ into a closed singular term denoting a relation. We have *the property of F-ing* if F is a verb, *the property of being F* if F is an adjective or prepositional phrase, and *the property of being (an) F* if F is a noun, where the indefinite article must be inserted if F is a count noun, and otherwise omitted. In place of the word *property* we may use: *trait, attribute*, and perhaps others. Quine (1970, p. 68–9) implicitly makes light of the productivity of the process yielding these nominalizations, writing that

> one may say that it is a trait or attribute or property of a born seaman never to quail at the fury of the gale when all one means is that a born seaman never quails at the fury of the gale

Natural-language nominalizations cannot always be dismissed in this way, however, for property-terms may appear as subjects and objects of a variety of locutions besides those that reduce to first-order sentences lacking such reference.

If properties and relations, like classes, are objects, then they do not enter the case for (or against) higher-order logic. But Montague (1960) gave an interesting explication of one conception of properties and relations in terms of intensional abstraction over predicates. The property, say, of being a nice fellow is interpreted in this system by

$^\wedge\lambda x$(nice fellow(x)) or $^\wedge$nice fellow

the function that, for each possible world, has for its value the function that, for each object, yields the value True just in case that object is, with respect to that world, a nice fellow. Relations are handled similarly, though here Montague pointed out that English does not always have a natural means of expression. For syntactic reasons, we do not say 'the relation of loving the sister of', and must have recourse to something like 'the relation that holds between x and y when and only when x loves the sister of y'. There is a relation H that expresses the comprehension principle for properties, rather naturally rendered in English by 'have', so we may write

(12) $(\forall x)(H(x,^\wedge$nice fellow$) \leftrightarrow$ nice fellow$(x))$

and similarly for relations, so that 'John and Mary stand in the shorter-than relation' would amount to 'John is shorter than Mary'.

One obvious reason for holding that properties and relations are not objects is that the type-distinctions block Russell's paradox, which would immediately arise for consideration if properties and relations were objects. However, as discussed especially in Chierchia (1984) and Chierchia and Turner (1988), there are a number of reasons for wanting property- and relation-reference to be reference to objects. One of these is that otherwise ordinary predicates must be systematically ambiguous as to type. If we are to say with Plato that Socrates, and also the property of being good, are good, then 'good' must be a predicate of two different types; and it is easy to construct unbounded iterations of predicate types. As in the case of Hodes's construal of arithmetic, we appear to want a kind of unification that the type theory deprives us of.

Montague's conception does respond to the demand, pressed especially by Quine, for an account of identity of properties and relations: in the intensional system identity amounts to necessary coextensiveness.

On the other hand, in construing properties and relations as essentially predicative, Montague was led to identify the interpretations of what are obviously predicates—'to be bald', 'being bald'—with what go into the language as singular terms such as as 'the property of being bald' (Montague 1960, p. 161). This decision marks a signal departure from the natural assumption, amply supported by the syntax of language, that the interpretations of the nominalizations and the predicates are fundamentally different, since their distributions are disjoint.

5.   *Plural reference and quantification*

Boolos and more recently Lewis have taken up plural quantification as a means of interpreting second-order theories (Boolos 1984), or of articulating non-first-order theories to serve as foundations for set theory (Lewis 1991). Inversely, Schein (1986), Higginbotham and Schein (1989) and Schein (1993) have discussed interpreting plurals in terms of second-order logic. All these discussions commence by arguing against a popular alternative, namely the treatment of plural terms as referring to something like sets, classes, or properties. In this section I will review and to some degree refine these arguments, turning afterwards to Boolos's discussion.

By a *plural term* I will mean either a definite description in the plural or a conjunction of singular or plural terms. Thus 'Peter and Paul', 'the boys', 'the books on my shelf and the magazines on my shelf', 'Peter and the other boys' are all plural terms. There are plural terms that are only revealed as such after taking account of quantifying into them: thus 'every man and his dog' contains a plural term 'x and x's dog', which will be the subject of predication in a sentence like 'every man and his dog went hunting together'; but I will not consider these cases further here. The problem of plural reference is the problem of the reference of plural terms, and also of plural quantifications, as in 'some (of the) boys', 'all (of the) books on my shelf and magazines on my shelf', and the like. These quantifications are not exhaustive, since besides quantifiers like 'many' and the numerals we have conjunctions as in 'some of the boys and Peter'; but again I will restrict the domain to the simplest cases.

In the literature both in philosophy and in formal semantics, plural reference has often been taken to be reference to sets, classes, or properties, and plural quantification as quantification over sets, classes,

or properties. What matters is not so much that the reference is to, say, sets exactly, but rather that plural reference, on the views in question, is taken to be singular reference in grammatical disguise. Naturally, something must mediate between the reference of a definite plural, such as 'the books', and certain individual books. In the influential work of Link (1983), each individual book on my shelf is an 'atomic i-part' of the reference of the phrase 'the books on my shelf'. It is such a part just in virtue of being a book on my shelf, and nothing other than a book on my shelf is such a part. But as pointed out by Higginbotham and Schein (1989), it follows that for Link the schema (C) holds unrestrictedly:

(C) x is an atomic i-part of the Fs $\leftrightarrow F(x)$

and therefore that, on pain of Russell's paradox, the predicate 'is an atomic i-part of' cannot figure in the object-language. The argument can be strengthened dialectically, as in Lewis (1991), or more straightforwardly as follows. Consider all plural terms 'the Fs' constructible in our language; such a term exists for every nominal expression F with a count noun head, and by hypothesis has a reference provided that there are at least two Fs. Given a universe of discourse for our language, there will be some objects in it that are the reference of plural terms; call these the *plural objects*. Then every plural term refers to a plural object, if to anything at all. A logic for plurals will not be complete unless it allows distributive quantification and the use of plural terms as parts of predicates, as in (13) and (14):

(13)   Each of the Fs is G
(14) x is one of the Fs / x is among the Fs

Assuming that the phrase 'the Fs' functions in (13) and (14) just as it would function as the argument of other predicates, these locutions will be allowed for by positing a relation, call it R, such that if $R(x,y)$ then y is a plural object, and construing (13) and (14) as using this relation tacitly, so that they become (15) and (16):

(15)   For all x such that $R(x, \text{the } F\text{s})$, $G(x)$
(16) $R(x, \text{the } F\text{s})$

Moreover, provided that the presupposition that there are at least two Fs is satisfied, (13) is equivalent to (17) and (14) to (18):

(17)   For all x such that $F(x)$, $G(x)$
(18) $F(x)$

(The equivalence of (14) to (18) of course implies the equivalence of (13) to (17).) Because (19) is an instance of (16) and (20) is the corresponding instance of (18), (19) and (20) are equivalent, provided that for at least two y, $\neg R(y,y)$:

(19) R(the objects y such that $\neg R(y,y)$), the objects y such that $\neg R(y,y)$)

(20) $\neg R$(the objects y such that $\neg R(y,y)$), the objects y such that $\neg R(y,y)$))

But everything that is not a plural object is a y such that $\neg R(y,y)$, so the presupposition is satisfied. And so we have Russell's paradox.

There are several points where the above deduction may be questioned; but none of them appear very promising.

(i)   Perhaps the plural term 'the objects y such that $\neg R(y,y)$' has no reference? But e.g. the books on my shelf are each of them things that are not plural objects, and so do not bear R to themselves. If the plural term has no reference, it must be that a meaningful predicate, satisfied by some objects, may fail to determine a plural object.

(ii)   Perhaps the expression (14) is not to be taken as in (16), but rather as a somewhat roundabout way of saying 'x is an F', schematically (18) itself; and similarly (13) is just a roundabout way of saying 'Each F is a G'? The definite article has a function, in that it delimits the universe of discourse—'three of the boys are here' is felicitous only if the boys in question are antecedently identifiable, whereas 'three boys are here' is at best neutral—but this difference is a matter of proper assertion rather than logical form.

This second way out amounts to denying that plural reference exists at all. Now, what motivated plural reference in the first place were cases where the plural appeared not to refer to a single object, whose parts in some appropriate sense were the Fs, but rather to the Fs taken somehow collectively, as could be seen from the fact that the distributional interpretations were intuitively false. These cases are matched by those where the form is just as in (14). For example, we do not regard

(21) x is one of the boys who built the boat

as equivalent to

(22) x is a boy who built the boat

Similarly for forms (13), since

(23)   Each of the boys who built the boat got a merit badge

is not regarded as equivalent to

(24)   Each boy who built the boat got a merit badge

The point may be put more sharply by considering simple uses of plural pronouns or demonstratives, without antecedents in a discourse. If I say, indicating some boys, 'three of them built a boat', then even if my utterance is understood as requiring and intending completion by a sortal, so that I am understood as talking about *those boys*, there need be no particular complete description 'the boys such that F' that I intend to communicate, so that the predicate that applies to each of them is just: 'x is among those boys'. If 'those boys' refers to a plural object, then some relation must mediate between this term and 'x'.

   (iii)   Perhaps the equivalence of (16) to (18) is not unrestricted? But on the contrary, it seems to be completely without exception, unimpeded by the question whether 'the Fs', on the construal suggested, form a set or something like a proper class. In fact, 'x is one of the proper classes' strikes us as equivalent to 'x is a proper class', and 'x is one of the plural objects' as equivalent to 'x is a plural object'. (This point, originally due to Boolos, is made in Lewis 1991 and Schein 1993, chapter 2.)

   The paradox of plurality just sketched seems to have a different status from the paradoxes of set theory and from the semantic paradoxes. In set theory we are not working within a given linguistic and theoretical scheme, so that the paradoxes can arguably be regarded as solved if there is some way out that allows theory to proceed. The semantic paradoxes arise naively, inasmuch as the concept of truth is not a technical one advanced for some theoretical purpose but a part of everyday vocabulary, and the general disquotational principles that give rise to the liar and related paradoxes reflect common linguistic practice. Plurality is like truth in appearing to be antecendently given, but the extra material that we brought in to explain the behavior of plurals, namely the hypothesis of plural objects and the relation R, are theoretical. All of this suggests that it is not the principles, whatever they are, governing the transition from 'x is among the Fs' to 'x is an F' and conversely that are to be faulted, but rather the hypothesis of plural objects that required positing R.

   Boolos (1984) used plural quantification to interpret monadic second-order formulas in set theory. He does not provide a theory of the

reference of plural terms such as 'the Cheerios in the bowl' (his example), but does hold that their use in making ordinary assertions does not augment the ontological commitments of our theory of the world. It is not easy to evaluate his thesis with respect to examples like this, however, because the Cheerios in the bowl are finite, indeed not many, so that it is hard at least for those of us who admit small finite sets to see what objection there could be to an ontology that admitted the set of Cheerios in the bowl; also because ontological commitment, if borne by plurals, would apply first of all to existential statements with plural quantificational prefixes. Putting the question of ontological commitment to one side, let me note by way of fixing ideas that Boolos's translations take as primitive the notion

(25)   it (or *x*; or *who, which* etc.) is one of them

where the singular and plural pronominals are both variables, indicating positions related to antecedents. Consider a second-order sentence such as

(26) $(\exists F)(\exists y)(\exists w)(F(y) \,\&\, F(w) \,\&\, y \neq w \,\&\, (\forall x)(F(x) \rightarrow (\exists ! z)$
       $(z$ is a parent of $x \,\&\, \neg F(z))$

Let the universe of discourse be people. In standard terms, (26) is true if a nonempty F can be chosen that is true of people of exactly one of whose parents it is not true. (Naturally, there are many such F.) Under Boolos's translation this becomes

(27)   There are some people such that every person who is one
       of them has exactly one parent who is not one of them.

The truth of (26) can be defended in second-orderese by letting F be true of me and my female ancestors, and no one else. Likewise, (27) can be verified by letting the quantifier 'some people' have (as one might say) for its value just: me and my female ancestors.

There are some limitations on Boolos's construction, which I will not consider here. The point of the construction, that monadic second-order logic is vindicated insofar as the translations into pretty ordinary English are well understood, is a powerful consideration in favor of second-order logic, whatever the peculiarities of natural language might otherwise be. With this much in mind, I want to consider plurals and plural quantification in themselves, so apart from the particular expressions that may be used to paraphrase second-order schemata.

6. *Interpreting plural quantification*

The instantiation of second-order schemata by means of plural quantification raises the question how plural quantification itself should be understood, and this in turn leads to the more fundamental question how ordinary definite plurals ('the books'), indefinite plurals ('some books') and bare plurals ('books') are interpreted in the sentences in which they occur. Schein (1986), Higginbotham and Schein (1989), and far more elaborately Schein (1993) defend the view that plurals in fact involve second-order quantification. That does not of itself undermine the interpretation of second-order quantification by means of plurals, because it may be that the sentences of the metalanguage, unpacking as they do the truth conditions of plural sentences, can in turn be instanced in English by means of plural quantification, again interpreted by means of second-order quantification, provided anyway that the interpretations are transparently equivalent. It could, however, be taken to mean that basing second-order logic on plural quantification is not fundamentally distinct from basing it on predication.

The fundamental problem of plural interpretation is the interpretation of undistributed plurals, as in the most salient interpretation of

(28) The boys built a boat
(29) Peter and Paul built a boat

We may be prepared to assert (28) without being prepared to assert of any particular boy that he built a boat, or (29) without being prepared to assert that Peter built a boat. Indeed we may correct a person's assertion that Peter built a boat by saying 'No, Peter and Paul built a boat'. Rejecting as we have plural objects (whether thought of as constituted, like sets, by their members, or in some other way), and assuming that the case is not one where a plural is used to refer to a single thing of which the various elements mentioned (Peter and Paul) or the things satisfying the predicate (those in the range of 'boy' in the context of utterance) are parts, this feature of our behaviour is not trivially explained.

I think that there are locutions in which definite plural NPs have singular reference. As discussed in Moltmann (1996, following Simons 1987), there are cases where the reference is to a single thing, of which the objects falling under the plural predicate are taken to be parts, in a contextually determined sense of that notion. To say, for example, that

the boxes together are heavy is to say that a certain single thing, the boxes together (or: taken together), of which the individual boxes are the parts, is heavy; to say that the books on the shelf are arranged alphabetically is to say that a certain thing, the family of books, of which the individual books are the parts, lies in a certain arrangement. Many cases, however, are not plausibly treated in this way: when I hear the children crying I do not hear the crying of a single object of which the chidren are the parts; rather, I hear an episode of crying to which various of the children make their contributions, or so it would appear. Furthermore, there are predicates that cannot be true of individual objects at all, such as 'clump' (Schein's example), 'rain down' (Boolos), 'cluster', and perhaps the reciprocal intransitives 'fight', 'meet', 'collide', and others.

If plural reference and quantification are taken as primitive then, as we might say, we can have predicates true of the many as well as of the one, and of course relations betwen the many and the one, as between the many boys and the one boat they built. But if we consider the logical form of plural sentences from a more elaborated point of view (as in Schein 1986, 1993), plural arguments undistributed with respect to the surface predicates may be reconstrued as distributed with respect to other predicates analytically posited.

To this end, consider the widely known thesis of Donald Davidson that action verbs contain a position for events, and suppose this thesis extended (as in Higginbotham 1985, Parsons 1989 and others) to all predicates whatsoever. Assume furthermore that the primitive predicates (nouns, adjectives, verbs, and prepositions) are related to their arguments through a family of specific relations, including the familiar relations of grammatical theory: 'agent,' 'patient,' 'beneficiary' and so on. These relations (known in contemporary generative grammar as 'thematic roles', though not always thought of as having the semantic content associated with them here) are borne by the arguments of the primitive predicate to the events, or more generally events and states, over which they range. The result is a picture of the simplest predication that natural language makes available as containing a complex structure, so that the skeleton for 'John walked', for example, would be as in (30):

(30) $\text{walk}(e)$ & $\Theta(\text{John},e)$

where 'walk' is a predicate of events, and Θ relates the event to its participant John, the reference of the subject. Sentences like 'John walked' are completed by adding tense, not considered here, and existential quantification, producing the familiar Davidsonian paraphrase

(31) $(\exists e)(\text{walk}(e)\ \&\ \Theta(\text{John},e))$

More generally, the picture is what might be described as a planetary theory of thematic roles: the head verb, or other primitive, is true of events whose participants are grouped around it like planets around a sun, attached by the various relations Θ. In a typical open sentence, 'x saw y' say, we have

$$(32)\quad x \xleftarrow{\quad\Theta_1\quad} \text{E=see} \xrightarrow{\quad\Theta_2\quad} y$$

Could not one and the same relation Θ relate each of several participants to the same event? Nothing in the picture prevents that, and so alongside the simple (32) we could also have

$$(33)\quad \begin{array}{c} x_1 \\ x_2 \end{array} \xleftarrow{\quad\Theta_1\quad} \text{E=see} \xrightarrow{\quad\Theta_2\quad} y$$

On this view, when we say that Peter and Paul built a boat we do not say anything about a complex agent, but rather aim to report an event of boat-building whose agents were Peter and Paul. We thus have for (29) the structure

(34)   built a boat(e) & $\Theta(\text{Peter},e)$ & $\Theta(\text{Paul},e)$

(I ignore the further structure that would come from the analysis of the predicate 'built a boat'.)

It remains to complete the structure by binding the event-variable in (34). But here a logical point must also be addressed. We do not want (29) to imply that Peter built a boat. But of course it will do so if the latter is just

(35) $(\exists e)(\text{built a boat}(e)\ \&\ \Theta(\text{Peter},e))$

What we require is that to say that Peter built a boat is not merely to say that there was an event e of boat-building of which Peter was an agent (i.e. to which he was related by Θ), but that there was a boat-building of which he was the *sole* agent, as in

(36) $(\exists e)(\text{built a boat}(e)\ \&\ (\forall x)(\Theta(x,e) \leftrightarrow x = \text{Peter}))$

Similarly, (29) is understood ultimately as

(37) $(\exists e)(\text{built a boat}(e)\ \&\ (\forall x)(\Theta(x,e) \leftrightarrow x = \text{Peter}\ \text{v}\ x = \text{Paul}))$

With this much to hand, we can proceed to the more complex case exemplified by

>   (38)   Some boys built a boat

In full, it will be

>   (39) $(\exists e)$(built a boat(e) & $(\exists X)(\exists x)(\exists y)(X(x)$ & $(X(y)$ & $x \neq y)$
>   & $(\forall z)(X(z) \rightarrow \text{boy}(z))$ & $(\forall w)(\Theta(w,e) \leftrightarrow X(w))$

Thus far I have been following Schein, who shows a number of further applications of his basic idea. Let us introduce the prefix 'there are some things whose number is at least 2, and . . .' for $(\exists X)(\exists x)(\exists y)(X(x)$ & $X(y)$ & $x \neq y$ & . . .) Then we may, following Boolos, translate (39) back into English syntactic form

>   (40)   There was a boat-building and there were some things
>   whose number is at least 2, all of whom were boys,
>   such that everything was an agent of it if and only if it
>   was one of them

Such harmony between the interpretation of second-order logic using plural constructions on the one hand, and the interpretation of plural constructions in terms of second-order quantification on the other, shows, pending examples to the contrary, that so far as we leave in place Boolos's locution 'it is one of them' we will not have a conflict between the two perspectives. I want now to argue, however, that plurals and plural quantification should be taken in the much older terms of the 'class as many', in the sense adumbrated by Russell (1903), rather than in terms of second-order logic. (Parsons 1990 notes the affinity between Boolos's interpretation and the class as many.)

7.   Classes as many

There are two rather obvious difficulties with the interpretation of plurals in second-order terms. The first is that the second-order logical forms considered to this point do not really allow primitive predicates with plural arguments; rather, any undistributed plural is taken to be distributed with respect to some relation, to one term of which the plural corresponds. In a statement like *these are a few of my favourite things* we do not appear to rely upon any mediating situations or relations to say that it is true if *these* are (at least) a few, and each of *them* is a favourite

thing of mine. In the case of *these are at least a few*, it apears that the numerical predicate is true of the reference of *these*, whatever it may be. And in the case of *each of them is a favourite thing of mine*, the question is how the quantification works—it appears to be universal quantification over objects x standing in some relation to the reference of the necessarily undistributed plural *them*.

The second difficulty impresses itself with respect to demonstrative undistributed plurals. Suppose I wave my hand at some boys, saying 'They built a boat yesterday.' I then make a definite reference of some sort—but to what? Adhering strictly to the second-order theory, one would take 'they' as answering to a predicate demonstrative, not a bound variable, as in

(41) $(\exists e)$(built a boat(e) & $(\forall x)(\Theta(x,e) \leftrightarrow A(x))$

where A just *is* what I referred to with the demonstrative. But there seems to be nothing predicational about the plural demonstrative.

The above difficulties are overcome if we regard plurals as referring to classes as many. We can enlarge standard semantics for English so as not to disturb the insight that some undistributed plurals express, not the relation of a single object to an event but rather the multiplicity of the objects that stand in that relation; for example, not group agency of some sort but multiple agency. But the axioms governing plural reference will themselves use plurals, so that undistributed plurals will appear in the specification of the semantics. An initial axiomatization for a relatively untendentious fragment of English would, I think, include the following:

(42)   for any singular nominal F, 'the Fs' refers to the Fs if there are at least two x such that F is true of x

(43)   For any singular or plural terms α and β, 'α and β' refers to α and β

In (43) the plural 'α and β' that is the object of 'refers to' is irreducible; that is, 'α and β' is not said to refer to α and to β, but rather to (α and β). We can formulate laws for terms such as Boolos's 'is one of', which takes one singular and one plural argument as in

(44) x is one of the Fs $\leftrightarrow F(x)$[1]

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[1] Exceptions occur when the context F allows only plurals. I think that most if not all speakers would reject a sentence like 'John is one of the people who love each other'.

(45) If α is a singular term and $x = \alpha$, or β is a singular term and $x = \beta$, or α is a plural term 'τ and δ' and x is one of τ and δ, or β is a plural term 'ε and μ' and x is one of ε and μ, or α is a plural term 'the Fs' and x is one of the Fs, or β is a plural term 'the Gs' and x is one of the Gs then x is one of α and β

To develop a full semantics for plurals as referring to the class as many, the assignments of values to variables will be construed plurally; that is, an assignment to a variable occupying a position where plural terms could go would itself have to be expressed plurally as, for instance, $s({}'x{}') =$ Peter and Paul (or Paul and Peter), and not as the unordered pair {Peter,Paul}. Similarly for the ranges of quantifiers: the quantifier 'some people' for a universe of discourse containing just Peter, Paul and Mary ranges over Peter and Paul; Peter and Mary; Paul and Mary; Peter, Paul and Mary; and nothing (no things) else.

The term 'is one of' is the plural ersatz for membership, and in the presence of plural quantification and plural variables it is not going to be eliminable. However, we do not lose the advantages of Schein's fundamental thesis, that undistributed plural reference may involve, not group agency, but multiple agency of single events. Using capital Greek letters for variables ranging over classes as many, examples like (46), reproduced here, will be as in (47):

(46) Some boys built a boat

(47) $(\exists e)$(built a boat(e) & $(\exists \Gamma)[(\exists x)(\exists y)(x$ is one of Γ & y is one of Γ & $x \neq y)$ & $(\forall z)(z$ is one of $\Gamma \rightarrow$ boy$(z))$ & $(\forall w)(\Theta(w,e) \leftrightarrow w$ is one of $\Gamma)]$

The language is essentially two-sorted; that is, there are contexts in which only plural, or only singular, terms (including variables) are admitted, one of them being the argument positions of 'is one of' itself. For this reason, attempts to reproduce Russell's paradox should founder on ungrammaticality. Thus

*the things that are one of themselves

The ersatz subset relation, call it 'Γ are included in Σ', is definable in terms of 'is one of' in the expected way: some things are included in some other things if everything that is one of the former is one of the latter, i.e.

(48) Γ are included in $\Sigma \leftrightarrow (\forall x)(x$ is one of $\Gamma \rightarrow x$ is one of $\Sigma)$

There are a number of points to be clarified, and even decisions to be taken, for a full two-sorted language of singulars and plurals. For one thing, I have omitted discussion of apparently plural terms like 'Bill Clinton and the President' that fail to be in order because their singular components are coreferential, or 'the boys and the members of the band', where some of the members of the band are boys. Also, our language is not *rigidly* two-sorted with respect to singulars and undistributed plurals, since many univocal predicates can accommodate both. Thus one can say

>   (49)   The soldiers surrounded the palace
>   (50)   The picket fence surrounded the palace

We could think of the reference of 'the soldiers' in (49) as to a single thing whose parts are the soldiers, much as 'the picket fence' refers to a single thing whose parts are the pickets and the pieces connecting them. But it may be preferable to think of the reference as to the class as many, which in virtue of having its elements spatially arranged in a certain way, surrounds the palace.

8. Commitments

If plurals are taken in the way that I have sketched, the question arises how if at all the notions of ontology and ontological commitment are to be applied to theories that use plural singular terms and variables. David Lewis (1991) has formulated a basis for set theory using mereology and plural quantification. The interpretation of plurals that I have suggested here is not, I believe, one that would suit his purposes. He takes plural quantification to be, as he says, *'sui generis'* and ontologically 'innocent'. Plural quantification is a special mode of quantification over what there is, namely objects. Mereology is also ontologically innocent, in his view, so that

> The fusion [of cats] is nothing over and above the cats that compose it. It just *is* them. They just *are* it. Take them together or take them separately, the cats are the same portion of Reality either way. (Lewis 1991, p. 81).

If Felix and Possum are two cats, and Felix + Possum is their fusion, then on the class-as-many view of plurals the words *Felix and Possum* refer to Felix and Possum (and so Felix and Possum are two), whereas *Felix + Possum* is a singular term (and so Felix + Possum is one). So

Felix and Possum ≠ Felix + Possum. So *they* are not *it*, contrary to Lewis's declaration. One may of course add that the fusion of Felix + Possum is nothing 'over and above' Felix and Possum, or vice versa; but the question would then be whether that statement expresses anything more than one's conviction that there is nothing ontologically committing about fusions, or plurals.

In sum, I have argued that plural reference and quantification are subject to analysis that deprives them of ontological innocence, and also of being a satisfactory basis for motivating second-order logic. But is the distinction between quantification over predicate positions and quantification over classes as many a 'shadow of grammatical distinction', as Charles Parsons once put it, or is it, as it was for Frege, 'written deeply into the nature of things?' Considerations such as I have given here are not decisive.[2]

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